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STEM Integration:
Making Connections in Mathematics and Science
by Teaching Logarithms Conceptually

By
Andrew C. Smith

A Dissertation
Submitted in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Education
In
Secondary Education
Mathematics

Kennesaw State University

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Dedication

For my wife, Jessica Smith, whose support, encouragement, and love have helped me to strive for excellence.

Acknowledgement

I would like to thank several people who have encouraged, motivated, and helped me through this journey. I would like to thank my committee chair, Dr. David Glassmeyer, for providing me encouragement and support throughout the process. I would also like to thank him for allowing me to participate with him in research related to my dissertation so that I may gain valuable experience to implement and write about my study. I would also like to thank my committee members, Dr. Kimberly Cortes and Dr. Kimberly Gardner, for taking time to mentor me through and providing valuable feedback for my dissertation. Thank you, Dr. Cortes, for asking the tough questions and providing feedback that would make me think and write more scholarly. Thank you, Dr. Gardner, for providing me an opportunity to share my work during your summer professional development program and guiding me through quantitative analysis.

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Abstract

The purpose of this research study was to examine both qualitatively and quantitatively the difference in conceptual understanding of logarithms of students participating in a traditional classroom setting in which a correspondence approach (Confrey & Smith, 1995) was used and students participating in an integrated STEM unit of logarithms and pH in which a covariational approach (Ferrari-Escolá, Martínez-Sierra, & Méndez-Guevara, 2016) was used. In addition, the researcher investigated how students make connections among different representations of logarithms and transferring knowledge between mathematics and science.

A quasi-experimental design was used in which qualitative data were collected using an Observation Protocol and quantitative data were collected using the Logarithms and pH Assessment (LPA). The qualitative data showed that students in the treatment group were thinking of logarithms in a mathematics and science context at a deeper level of conceptual understanding according to Weber's (2002) levels of understanding exponential and logarithmic functions and Park and Choi's (2012) levels of understanding pH. In addition, the qualitative data showed that students in the integrated STEM classroom were better able to transfer their knowledge of logarithms to pH and make deeper connections among different representations of logarithms (numerical, algebraic, and graphical). However, the quantitative data from the LPA, which defines conceptual understanding as the ability to represent concept in multiple forms: written, numerical, algebraic, and graphical (Panasuk, 2010; Rittle-Johnson, Siegler, & Alibali, 2001), indicated that the traditional classroom developed students' conceptual understanding more than the integrated STEM classroom.

This study provides mathematics and science educators with relevant information about incorporating integrated STEM lessons and covariational reasoning to teach logarithms

conceptually, to improve a student's conceptual understanding of logarithms and pH, and their ability to apply mathematical knowledge to settings other than the mathematics classroom.

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Chapter 1: Introduction

Science, technology, engineering and mathematics (STEM) jobs have been defined as any occupation in computer science, mathematics, engineering, life science, and physical science (Langdon, McKittrick, Beede, Khan, & Doms, 2011), and their availability is on the rise. Currently, about one out of six jobs are considered STEM, and though STEM jobs are not a large portion of the job market, the rate of growth of STEM jobs is about double the rate of growth of non-STEM jobs (Langdon et al., 2011). Therefore, the United States economy is becoming more dependent on STEM education for developing skills necessary for STEM jobs (Capraro & Nite, 2014). However, college students are not enrolling in STEM courses to keep up with the growth of STEM jobs (Capraro & Nite, 2014), and students pursuing STEM degrees have the highest attrition rate (Drew, 2011). Two of the reasons for this high attrition rate include a lack of math preparation and typically lower grades in STEM classes than in non-STEM classes (Drew, 2011).

Each of these reasons can be linked to a student's conceptual understanding of math skills. Panasuk (2010) describes conceptual understanding in algebra as a student's ability to recognize relationships and interpret different representations of those relationships. A student with conceptual understanding of a topic can demonstrate fluency in the use of algebraic vocabulary and operations and use multiple representations to describe concepts. In addition, Potgieter, Harding, and Engelbrecht (2008) describe conceptual understanding of mathematics as a student's ability to transfer their knowledge to other disciplines. For example, Leopold and Edgar (2008) found that students with a more developed conceptual understanding of the mathematics used in STEM were more successful on average in STEM courses. In addition, Park and Choi (2010) found that many students have difficulty conceptually understanding

scientific concepts, because they have little conceptual understanding of the mathematical concept involved. Therefore, a student's conceptual understanding of mathematical topics was a good predictor of success in a STEM class.

In addition, many teachers of mathematics find that the pressure of time and high-stakes testing drive them toward teaching rote memorization instead of conceptual understanding (Nagy, 2013). Students gain procedural knowledge and fail to develop any true understanding. As a result, educators should examine ways to develop a student's conceptual understanding within mathematics and science to increase a student's ability to be successful when pursuing a STEM degree. One such way is integrating mathematics and science topics, so students can learn and apply the topics of each discipline together. Research integrating mathematics and science has greatly increased since the 1980's (Berlin & Lee, 2005). Educators and researchers have noticed that though mathematics and science are two unique disciplines, the two subjects depend and reinforce each other (American Association of the Advancement of Science, 1994). In addition, the National Council of Teachers of Mathematics (NCTM) (2000) stated that students should be provided opportunities to learn and use mathematics in different context such as other subject areas and within their daily lives.

More recently, STEM education has become the term used to emphasize an integration of mathematics and science by using technology and engineering. However, STEM education is a broad term that can be defined in several ways (Reeve, 2015), and researchers have not clearly defined what constitutes quality STEM education. Tsupros, Kohler, and Hallinen (2009) provide a commonly cited definition and described STEM education as an interdisciplinary approach to learning. The researchers suggest that STEM education should make some connections among different content disciplines, but educators have often focused on individual subjects (Honey,

Pearson, & Schweingruber, 2014). Reeve (2015) refers to this as teaching in “silos”. However, advocates for STEM education have called for an integrated approach (Honey et al., 2014). Educators should be teaching STEM content in a more connected manner by incorporating real-world context that will make the STEM subjects more relevant and connected.

Statement of the Problem

Logarithms and pH provide opportunities for educators to integrate mathematics and science to help develop a student’s conceptual understanding of both topics. Logarithms and pH are related but are taught in two different STEM disciplines at the high school level (Common Core Standards Initiative (CCSS), 2017; Next Generation Science Standards (NGSS), 2017). The NGSS call for students to conceptually understand acid and base reactions which will require students to have a conceptual understanding of pH. However, research has shown that students struggle with pH in post-secondary educational settings, because they have difficulties connecting their mathematical knowledge with science (Heckle, Mikula, & Rosenblatt, 2013; Hoban, Finlayson, & Nolan, 2013; Leopold & Edgar, 2008; Potgieter et al., 2008). Therefore, students could benefit if they are able to develop a deeper level of understanding through an integrated STEM lesson when they are initially exposed to logarithms and pH in high school. Students may be able to apply these topics to future endeavors, if they can first build their conceptual understanding through an integrated STEM curriculum.

Purpose of Study

The purpose of this research study was to examine the difference in conceptual understanding of students participating in a traditional classroom setting and students participating in an integrated STEM unit. The integrated STEM unit connected the concepts of logarithms from Algebra II and pH from chemistry, while the traditional classroom setting

focused strictly on logarithms in mathematics and strictly on pH in chemistry. In other words, students participating in an integrated STEM unit were explicitly taught logarithms and pH in both mathematics and chemistry classrooms while students participating in a traditional classroom setting were implicitly taught logarithms in science and implicitly taught pH in mathematics.

Significance of the Study

This study provides educators with relevant information about teaching logarithms conceptually and incorporating integrated STEM lessons to improve a student's conceptual understanding and ability to apply mathematical knowledge to settings other than the mathematics classroom. Research has shown that students struggle with applying their mathematical knowledge outside of the mathematics classroom (Heckle et al., 2013; Hoban et al, 2013; Leopold & Edgar, 2008; Potgieter et al., 2008), and the research also states that students struggle with understanding the scientific concepts because of a lack of conceptual understanding of the underlying mathematical content. Therefore, this study provides relevant information to help educators teach conceptually to make a student's mathematical knowledge more flexible, so they can apply their knowledge to multiple and unique real-life applications.

Chapter 2: Literature Review

STEM education is a broad term and researchers have not clearly defined what constitutes quality STEM education (Reeve, 2015). Likewise, integrated STEM education or STEM integration is a broad term with several definitions (Reeve, 2015) and methods of implementation (Hurley, 2001). Hurley defined five ways to integrate mathematics and science: sequenced, parallel, partial, enhanced, and total (Table 1).

Table 1

<i>Types of mathematics and science integration</i>	
Type	Description
Sequenced	“Science and mathematics are planned and taught sequentially, with one preceding the other.
Parallel	“Science and mathematics are planned and taught simultaneously through parallel concepts.”
Partial	“Science and mathematics are3 planned and taught partially together and partially as separate disciplines in the same classes.”
Enhanced	“Either science or mathematics is the major discipline of instruction, with the other discipline apparent throughout the instruction.”
Total	“Science and mathematics are taught together in intended equality.”

Note: From “Reviewing integrated science and mathematics: The search for evidence and definitions from new perspectives” by M. M. Hurley, 2001 *School Science and Mathematics*, 101(5), 259-268.

From these five methods, total integration was found to have the most impact on both science and mathematics. This type of integration provides lessons that treat each integrated discipline with equal importance. In a total integrated approach, mathematics and science can be taught more explicitly within each discipline and lessons should be planned collaboratively within each course. Therefore, quality STEM integration of mathematics and science should incorporate a total integrated approach.

Researchers incorporating different STEM integration models show positive results when emphasizing the importance of each STEM discipline (Berland, Steingut, & Ko, 2014, Guzey,

Moore, & Harwell, 2016, Stone, Alfeld, Pearson, Lewis, Jensen, 2006, Wang, Moore, Roehrig, & Park, 2011), but these studies show flaws in implementing STEM integration at the high school level. They found that students without mathematics and science support do not value the mathematics or science involved. They found that educators need support with the STEM subjects in which they are less familiar when designing STEM integrated lessons and that educators need support throughout the implementation process in STEM disciplines in which they are less familiar. Therefore, this study describes a proper STEM integrated lesson as a lesson that explicitly teaches the content from each integrated discipline, involves educators from multiple disciplines in planning, and continuously involves teachers from multiple disciplines throughout the implementation.

First, quality STEM integration incorporates real-world problems, so students can understand the importance of each STEM discipline. Real-world problems from daily life often involve multiple STEM disciplines, which suggest that STEM integration is an appropriate and effective way of learning (Capraro & Nite, 2014, Honey et al., 2014). In addition, real-world problems make the learning more relevant for the students, and problem-based learning (PBL) can help students develop critical thinking and problem-solving strategies through challenging real-world problems (Reeve, 2015). However, PBL is not enough to ensure that high school students are meeting the intended goal of the lesson. For example, Berland et al. (2014) studied how students engage in the engineering design process: defining a problem, developing multiple solutions, modeling and analysis, and iteration. They used *Engineer Your World* curriculum that was designed to integrate multiple STEM disciplines through the engineering design process. Students ranging from sophomores to seniors in high school participated in six units that required them to reverse engineer, design and build, and analyze data. The units were administered in a

PBL environment, which is a student-centered environment in which students are required to gain knowledge through self-directed learning (Barrow, 1996). Berland et al. (2014) found that students were more comfortable with the qualitative aspects of design. Students did not think about the projects in a scientific or mathematical manner. They worried more about the appearance than the workability of the design. In addition, the researcher found students did generate multiple solutions to the projects, but they did not use a systematic way of deciding between them. As a result, the researchers concluded that students mainly value the parts of the engineering design process that do not require them to integrate their mathematics or science content knowledge.

The intention of Berland et al. (2014) was to investigate students use of the engineering design process, but the researchers show, that in a PBL setting, students tend to undervalue the importance and fail to use their knowledge of mathematics and science. Therefore, the units that were taught could be improved by providing the students with the appropriate support and guidance to lead them toward an integration of math and science. PBL creates opportunities for students to integrate math and science and apply the knowledge to real-world situations (Berland et al., 2014), but high school students do not automatically make connections among prior knowledge (Honey et al., 2014). Experts make connections among ideas and understand the usefulness and importance of their knowledge to inform those ideas. Students or novice learners need guidance and explicit support to make those connections (Honey et al., 2014). Therefore, creators of STEM integrated lessons for high school should make sure that a scaffold and more explicit approach to connecting multiple STEM disciplines is used.

Second, quality STEM integration identifies compatible standards from multiple disciplines that can be incorporated into an integrated lesson (Capraro & Nite, 2014). Students

are required to make connections among different disciplines and develop both a concrete and abstract knowledge of each STEM discipline involved (Honey et al., 2014). The quality of a STEM integrated lesson starts with careful planning and collaboration among educators. However, collaboration does not ensure properly developed lessons. For example, Guzey et al. (2016) used a professional development (PD) program to help teachers design and implement integrated STEM units. About 50 science teachers participated in the PD and were challenged with the task to create an integrated STEM unit. The teachers teamed up with other science teachers and were provided a team coach, who had previous experience designing integrated STEM lessons. In total, the teachers designed 20 integrated STEM units. However, less than half of the units adequately integrated mathematics and science. The researchers concluded that science teachers have difficulties integrating mathematics, because they have limited knowledge and lack the ability to teach it effectively. As a result, teachers focused on the engineering and science integration while possibly choosing to ignore the mathematics in the curriculum.

Teachers need opportunities to learn and practice STEM disciplines, and they must be provided support with the STEM disciplines in which they are less familiar (Guzey et al., 2016). Guzey et al. did not provide an adequate amount of support during the planning process. Teachers designed integrated STEM units that lacked certain key disciplines in STEM. Sanders (2009) states that it would be difficult to expect an individual to possess enough content knowledge to teach all STEM disciplines effectively. Though Guzey et al. allowed teachers to form teams and design units; only science teachers were involved in the process. Teachers need to collaborate with other educators from different disciplines to effectively design an integrated STEM unit. If a quality integrated STEM unit is to be created, teachers from multiple disciplines

should be involved in the planning process. For example, educators in both math and science should be involved if the STEM unit integrates both math and science.

Last, quality STEM integration involves collaboration among teachers from different STEM disciplines throughout the implementation of an integrated STEM unit. A challenge that educators face when implementing an integrated STEM unit is confidence in teaching content from multiple disciplines (Honey et al., 2014). Teachers need to collaborate with other content experts to ensure the quality of the integrated STEM unit. For example, Stone et al. (2006) hypothesized that a student's conceptual understanding and transferability of math skills could be enhanced by using an integrated approach. They believed that if math is taught more explicitly within the Career and Technical Education (CTE) setting, then students will improve their conceptual understanding and transferability of basic mathematical content. In other words, students will be able to apply their mathematical knowledge to multiple areas and the students' mathematical ability will become more general and less specific. The researches recruited volunteer CTE teachers to participate in designing and teaching math-enhanced lessons during the CTE course. Each CTE teacher was paired with a math educator to help identify mathematical content, build the lessons, and help the CTE teachers to re-learn the mathematical concepts involved in the lessons. The math-enhanced lessons were designed to more explicitly teach math content already existing in the CTE curriculum. Before the units were implemented, the CTE and math partners met to refresh the CTE teacher's knowledge on the mathematical content involved. The partners did not meet again until after the lesson was taught. The researchers found that students could improve their conceptual understanding of mathematics with no effect on their ability to learn the CTE content. Students could transfer mathematical knowledge to the CTE context when math was explicitly taught within the CTE setting.

However, Stone et al. (2006) did state that one challenge that arose with a more explicit approach to mathematics in the CTE setting. CTE teachers are not math teachers, and therefore they are not the most effective at teaching mathematics. Some CTE educators could not teach basic algebraic concepts without remediation.

Like Stone et al. (2006), Burghardt, Hecht, Russo, Lauckhardt, and Hacker (2010) studied a way to enhance Engineering/Technology Education (ETE) by infusing mathematical content. The researchers felt that mathematics is the link between STEM disciplines and infusing the curriculum with mathematics was a meaningful way of making connections among STEM disciplines. Unlike Stone et al., Burghardt et al. did not require the participating educator to design the unit. They used an existing research-based unit called the Bedroom Design Activity, which is a 20-day unit that engages the students in planning, designing, and modeling a bedroom that must meet specific measurements and budgets. In addition, the researchers required each participating teacher to attend PD that would guide, mentor, and train each on the implementation of the unit. The researchers claimed that the PD would ensure that the participating teachers were familiar with the content and pedagogical knowledge necessary to teach the unit. The researchers found that not all mathematical content that was infused showed a statistically significant improvement on student achievement. They indicated that, even with a rigorous PD, a possible cause for this disparity was due to ETE teachers struggling with teaching the mathematical content that was infused.

Unlike, Guzey et al. (2016), Stone et al. (2006) and Burghardt et al. (2010) did provide content experts during the planning process to possibly help bolster the quality of the integration of mathematics in the lessons, but the support was not continued during the lesson. Teachers of non-mathematics courses should not be expected to teach mathematics well (Sander, 2009).

Therefore, to properly integrate STEM disciplines, teachers from at least two content areas need to collaborate during the planning process and during the lesson. Though both need knowledge in each STEM discipline, each teacher can rely on the other to teach domain specific content. As a result, students could improve their ability to transfer knowledge by learning each topic from the content expert.

Therefore, a proper STEM integrated unit must teach mathematics and science explicitly through PBL, have teachers from multiple STEM disciplines identify compatible standards and be involved in the planning, and must have teachers from multiple STEM disciplines continuously involved in the implementation. In other words, a total integration model of mathematics and science should be used when designing an integrated STEM lesson. Experts make connections among ideas and understand the usefulness and importance of their knowledge to inform those ideas. Student or novice learners need guidance and explicit support to make those connections (Honey et al., 2014). Integration of STEM content should be made explicit by intentionally teaching multiple disciplines within the classroom. In addition, teachers should not be expected to be experts in multiple STEM disciplines (Sanders, 2009). Educators have the knowledge necessary to teach their specific content. However, a properly designed STEM integrated unit will contain content from multiple disciplines, and it will require content specific pedagogical knowledge to help guide the students in learning. Therefore, teachers should collaborate with educators from different disciplines to ensure that appropriate content is incorporated into the unit, and teachers should continuously collaborate throughout the implementation of the unit to ensure that students are taught properly.

Educators have the opportunity to incorporate an integrated STEM lesson using logarithms (mathematics) and pH (chemistry). Logarithms and pH are related but are taught in

two different STEM disciplines at the high school level in Georgia: Algebra II and Chemistry. In addition, students learn for the first time both logarithms and calculating pH using logarithms during those two courses (Georgia Standards of Excellence, 2015). Students could benefit by developing a deeper level of understanding when they initially are exposed to these topics. Students will be equipped to better apply these topics to their future pursuits in college or workforce if they can first build their conceptual understanding in high school.

For example, when studying college students' ability to transfer mathematical knowledge to chemistry, Hoban et al. (2013) found that the mathematical difficulties experienced by students were due to a lack of conceptual understanding of the mathematical topics. Students either lacked the skills to perform mathematical operations in a science context or were able to succeed due to becoming familiar with the process of using the mathematics in a chemistry context, which could be viewed as superficial knowledge and not true understanding. In addition, Leopold and Edgar (2008) state that one of the first college science classes in which difficulties with mathematics becomes a problem is second semester chemistry. One topic of this course is Acid-Base chemistry in which logarithms and pH are related. The researchers found that many students were familiar with the mathematical rules for manipulating logarithms, but many of the students did not understand the meaning of a logarithm. Again, many students were only able to use a superficial knowledge of a mathematics topic in the science context without a developed conceptual understanding. Likewise, Heckler et al. (2013) found that sophomore, junior, and senior level undergraduate engineering students had significant difficulties reading and interpreting logarithmic plots. Potgieter et al. (2008) state that this difficulty could be caused by mathematics educators teaching algebra and graphs as two separate entities and the lack of graphical representations in the science textbooks. These few examples

provide evidence that a significant portion of the student population that pursue STEM degrees are not gaining a conceptual understanding of logarithms in earlier mathematics courses, and therefore have difficulties applying logarithms to pH. In addition, Leopold and Edgar (2008) found that explicitly teaching mathematics in conjunction with science helped strengthen a student's math skills and therefore provided evidence that with a diligent work ethic one can excel in spite of mathematical deficiencies.

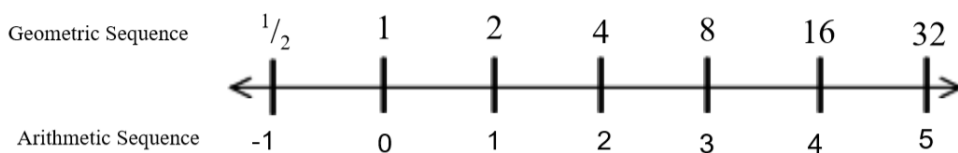
Logarithms and Students' Conceptual Understanding

Students at the high school level need to develop a "good understanding" of logarithms. According to the Georgia Standards of Excellence (2015), students need to be able to convert equations between an exponential form and a logarithmic form, graph and identify key characteristics of logarithms, be able to use properties of logarithms, and understand the inverse relationship between exponential and logarithmic functions. In addition, students need to know the common and natural logarithm. However, students can do the algebraic manipulation necessary to demonstrate knowledge of the standards without developing a "good understanding" (Hoban et al., 2013, Leopold & Edgar 2008).

Leopold and Edgar (2008) state when students are allowed to use a calculator they can mask their deficiency in their conceptual understanding and mental math skills. Therefore, students should be able to answer questions about logarithms without the use of a calculator to demonstrate a "good understanding." Students should be required to demonstrate their mental math skills by answering questions that require them to evaluate, compare, and estimate. For example, students could be asked to evaluate $\log 0.01$. Students would be expected to respond with an answer of negative two. Though this question can be answered with a memorized response, students will also be asked to compare logarithms. For example, which is greater

$\log_2 10$ or $\log_4 45$? Students would be expected to respond with $\log_2 10$. In addition, students will be asked to estimate by approximating logarithms such as $\log_5 50$. Students with a “good understanding” would be expected to answer the question with a value between 2.3 and 2.5, because $5 * 5 * \sqrt[3]{5} \approx 5^{2.3} < 50 < 5^{2.5} = 5 * 5 * \sqrt{5}$.

In addition, Ferrari-Escolá et al. (2016) state that simply knowing that a logarithmic function and exponential function are inverses does not demonstrate an understanding. The researchers believe that students should understand both the exponential and logarithmic functions as a juxtaposition of an arithmetic sequence (a sequence of numbers in which a constant is added to progress to the next term) and geometric sequence (a sequence of numbers in which a constant is multiplied to progress to the next term). For example, students should understand that adding in the arithmetic sequence corresponds to multiplying in the geometric sequence. Therefore, students should know that for every property of addition there is a corresponding property of multiplication. Confrey and Smith (1995) describe students’ connection between the arithmetic and geometric sequences as covariational reasoning, and Smith and Confrey (1994) define a logarithm as the corresponding number in an arithmetic sequence for a given value in the geometric sequence. For example, figure 1 shows that in a base two system the logarithm of 8 is the corresponding value in the arithmetic sequence, which would be 3.



$$\log_2(8) = 3$$

Figure 1. Example of Smith and Confrey (1994) definition of a logarithms

Confrey and Smith (1995) state that covariational reasoning is better than a traditional correspondence view of exponential and logarithmic function, which relates the domain to the range by an explicit rule. The correspondence view causes students to rely on algebraic manipulations and procedural skills (Confrey & Smith, 1995), unlike covariational reasoning which encourages students to reflect on their knowledge based on each condition (Ferrari-Escolá et al., 2016). Therefore, students can demonstrate “good understanding” by their ability to relate the properties of an arithmetic series to the properties in a geometric series (properties of logarithms). For example, students could be given the general equation $\log_b a = c$ and asked questions such as what happens to a when c is doubled? Students should respond with “ a will be squared, because repetitive addition in the arithmetic sequence corresponds to repetitive multiplication in the geometric sequence.” Other questions could include:

- What happens to “ a ” when 3 is added to “ c ”?
- If “ a ” is divided by “ b ”, what happens to “ c ”?

Furthermore, students should be able to interpret logarithmic graphs by identifying key characteristics to demonstrate a “good understanding” (Leopold & Edgar, 2008; Potgieter et al., 2008). Potgieter et al. state that algebra students without a visual understanding of the representation of a function have an inadequate understanding of the mathematical topic. Therefore, students should be asked to demonstrate a conceptual understanding of logarithmic graphs. They should be able to explain the meaning of the x-intercept of a logarithmic function of the form $f(x) = \log_b x$ as the identity of the geometric sequence (domain). In addition, students should be able to explain the meaning of the vertical asymptote as the lower limit of the domain. In other words, repetitive division in the geometric sequence will never reach a value of zero. Students should also be able to answer questions that compare rates of change. For

example, students should be able to state that the function $f(x) = 2^x$ is increasing as x increases, because repetitive multiplication of a number greater than one will result in a larger number.

In addition, Weber (2002) describes different levels of students understanding of exponential and logarithmic functions: exponentiation as an action, exponentiation as a process, exponential expression as the result of a process, and generalization (Table 2). Students at the “exponentiation as an action” level see exponents as an action of repetitive multiplication and can only evaluate exponential functions using positive integers. Students at the “exponentiation as a process” level have internalized that action from the previous level of understanding. They will be able to imagine results or understand properties without any calculation. For example, a student can understand that a positive number to any power is positive. In addition, students at the process level can also reverse the exponentiation and form the process of a logarithm. Students at the “exponential expressions as the result of a process” level view b^x as computing b times itself x number of times or as the product of x factors of b . At this level, students develop an understanding of the properties of exponents and logarithms. However, a student’s understanding of exponential and logarithmic functions is still limited to natural numbers. Students at the “generalization” level are able to interpret an exponential and logarithmic function for values that are negative, fractional, or irrational. Therefore, students possessing a “generalized” level of understanding should be able to answer questions that involve non-natural numbers. For example, students could be asked to estimate x in $\log_2 x = 0.5$. They should be able to provide an answer of “the square root of 2.” In addition, students could be asked to estimate the value of $\log_2 1.001$. They should provide an answer close to zero, because they understand that two raised to the zero power is one. Weber also emphasizes that students at the “generalized” level of understanding should be able to articulate complex ideas of exponential

and logarithmic function such has the half power without the use of given rules from a teacher or textbook.

Table 2

Stages of Conceptual Understanding of Exponential and Logarithmic Functions	
Stage	Evidence
Exponentiation as an action	<ul style="list-style-type: none"> • Evaluate exponential functions with only positive integers exponents. • Compute values and manipulate formulas
Exponentiation as a process	<ul style="list-style-type: none"> • Understand simple properties of exponential functions such as increasing or decreasing • understand the process of logarithms. • Restricted to natural numbers
Exponential expressions as the result of a process	<ul style="list-style-type: none"> • Can explain common properties of exponents and logarithms such as adding exponents when multiplying common bases • Restricted to natural numbers
Generalization	<ul style="list-style-type: none"> • Interpret exponents that are fractions, negative, or irrational without the use of a given rule from a textbook or teacher
<i>Note: From “Students’ Understanding of Exponential and Logarithmic Functions” by K. Weber (2002)</i>	

pH and Students’ Conceptual Understanding

Jackson, Johnson, and Blanksby (2014) state that mathematical competence is imperative for studying the sciences and increasing a student’s ability to use mathematics in the science context. Therefore, in addition to understanding logarithms in mathematics, students need to develop a “good understanding” of logarithms in science. According to the Georgia Standards of Excellence (2015), students need to be able to analyze the nature of an acid or a base by its percent of disassociation, hydronium ion concentration, and pH using mathematical thinking. In other words, students need to be able to understand the link between hydronium concentration and pH by using logarithms. Therefore, a meaningful understanding of pH is important to understanding logarithms in the science context (Park & Choi, 2010)

Park and Choi (2012) describe various levels of the conceptual understanding of pH: object, operation, function (Table 3). Students at the object level describe pH as a characteristic of a substance. Their understanding of pH is more descriptive. Students at the object level develop a basic knowledge of pH and commonly define pH as a measure of acidity. Students at the operation level are able to use procedural mathematics skills to obtain and describe a difference in pH, but they still define pH as a measure of acidity. These students are able to perform operations based on proportional reasoning either in the pH scale or in the logarithmic scale. For example, students at the operational level would calculate a difference between a pH of 1 and 3 by answering “3 times stronger” or “100 times stronger”. Though these students are able to provide a numerical difference between pH values, they do not see pH as a measure of the concentration of hydrogen ions. Students at the function level of understanding are able to connect hydrogen ion concentration to pH values. They see pH as a function that varies depending on the concentration of hydrogen ions. Therefore, they possess all the proportional reasoning skills present in the previous levels of understanding, but they understand that the pH is a function of hydrogen ion concentration. Therefore, a student that possesses a “good understanding” of logarithms in science (pH) sees logarithms as a function connecting two variables: hydrogen ion concentration and pH.

Table 3

Domains of students' understanding of the pH value		
Domain	Type	Evidence
Object	Type 1: Measure of Acidity	<ul style="list-style-type: none"> pH is a measure of acidity pH is a qualitative descriptor Differences in pH values is described qualitatively such as dangerous or safe
	Type 2: Measure with proportional differences	<ul style="list-style-type: none"> pH is a measure of acidity pH values are proportional to a base (pH of 1 and pH of 3 have a relative difference of 100) Base of 10 is memorized
Operation	Type 3: Measure with proportional differences in logarithms	<ul style="list-style-type: none"> pH is a measure of acidity can manipulate the logarithmic formula Obtain proportional differences Mathematical understanding disconnected from scientific meaning.
	Type 4: Concentration with proportional differences	<ul style="list-style-type: none"> PH is a measure of Hydrogen ion concentration pH values are proportional to a base (pH of 1 and pH of 3 have a relative difference of 100) Base of 10 is memorized
Function	Type 5: Concentration with proportional differences in logarithms	<ul style="list-style-type: none"> pH is a measure of Hydrogen ion concentration can manipulate the logarithmic formula Obtain proportional differences Mathematical understanding connected to scientific meaning.

Note: Form "Analysis of students understanding of science concept including mathematical representations: pH values and the relative difference of pH values." By E. Park & K. Choi, 2012 International Journal of Science and Mathematics Education.

Therefore, to assess a student's level of understanding of logarithms in the science context students will need to be asked questions that involve pH. First, students will need to answer questions about the meaning of pH (Park & Choi, 2010, Park & Choi, 2012). For example, Park and Choi (2012) used questions two through five in their interview protocol

(Appendix A) to assess a student's definition of pH. Students were asked to explain the definition, the purpose, and the range of pH. A student that shows a "good understanding" of logarithms in science will answer these questions by referring to pH as the concentration of hydrogen ions or hydronium ions in a solution. Another appropriate answer would be the formula $pH = -\log[H^+]$. However, the researchers warn that the writing of a formula does not guarantee a functional understanding of pH. Second, Park and Choi (2012) further investigated a student's understanding of pH by asking questions that involve proportional differences in pH (items 6-9). In these four items, students were asked to compare solutions with two different pH values, represent and explain the spacing of those values on a number line, and graph the pH versus the molarity of a solution. A student showing a "good understanding" of logarithms in the science context would understand that the difference in pH values is a power of ten. For example, if a student was asked to calculate the difference between a pH of 2 and a pH of 4, he or she would be able to understand that the pH of 2 has a concentration of hydrogen ions that is 100 times more than the solution with a pH of 4. In addition, a student comparing the pH of 2 to the pH of 4 could draw a number line based on the hydrogen ion concentration and show that the spacing between a pH of 2 and pH of 3 is more than the spacing between the pH of 3 and the pH of 4. In addition, students showing a "good understanding" of logarithms would graph a curve in which the molarity of the hydrogen ions decreases as the pH value increases.

Similarities and Differences between the Mathematics and Science Context

Math and science content are sometimes inseparable. Students must use knowledge from both to apply their skills to the real-world. For example, students have to engage with math and science to solve engineering problems (Berland et al., 2014). However, Jackson et al. (2014) state that a student's transfer of mathematical skills to science is not straightforward for him or

her. Therefore, both math and science teachers need to provide the students with consistent approaches to teaching mathematical concepts in mathematics and science classrooms. In addition, Jackson et al. state that the differences in language can create barriers that prevent students from transferring their knowledge from one STEM discipline to another. Therefore, educators need to look at the similarities and differences in teaching certain topics such as logarithms in mathematics and science classrooms and come to a consensus of how best to teach logarithms.

In mathematics, some teachers emphasize the abstract definition of a logarithmic function as the inverse of an exponential function and state that a logarithm is a link between x and $f(x)$. The focus in this approach is the explicit formula that links the domain to the range. To teach this definition of logarithms, Hamdan (2008) showed that in mathematics classrooms students first develop their knowledge of the properties of exponents. Then students develop a knowledge of exponential functions which includes various properties such as continuous and one-to-one. Because an exponential function is one-to-one, students then develop the idea that an exponential function is invertible. Then, students name the inverse function as the logarithm. Once students have an idea of logarithms, they begin to work with the algebraic properties of logarithms and characteristics of their graphs (Georgia Standards of Excellence, 2015).

In a science classroom, educators should link logarithms to pH. The pH value should be recognized by students as the measure of hydrogen ion concentration. However, Park and Choi (2012) stated that pH is frequently taught as a characteristic of a substance. As a result, most students see pH as a descriptive characteristic and logarithms are only a means to calculate pH. Sheppard (2006) stated that textbooks tend to focus on calculating the value of pH rather than a conceptual understanding of pH. Therefore, like mathematics, students can develop an

understanding of logarithms as a link between two variables by using an explicit rule. In addition, Sheppard stated that pH is taught within the acid-base unit in chemistry and due to the sheer size, time allocated, and amount of prior knowledge required, teachers typically use a traditional lecture model for this unit. Therefore, students rely on rote memorization and procedural skills to succeed in this unit. In addition, unlike mathematics, Potgieter et al. (2008) stated that science textbooks place little to no emphasis on the graphical representations of the mathematical content. As a result, teachers rely on the worked-examples to teach students to use logarithms to calculate pH and provide no visual representation of the logarithmic function.

Therefore, in both the science and mathematics classroom, teachers mostly use a correspondence approach (Confrey & Smith, 1995) when teaching logarithms. Students are taught that logarithms are a link between two variables. However, in mathematics, the two variables are abstract, but in science, the two variables are hydrogen ion concentration and pH. In addition, Potgieter et al. (2008) provided evidence that math and science teachers provide little to no emphasis on the connection between the algebraic and graphical nature of logarithms. In a science textbook, graphical representations are typically absent, and in a mathematics classroom, graphical and algebraic representations are taught as separate entities.

However, if logarithms are to be understood in both the science and mathematics classrooms, educators must collaborate. Park and Choi (2010) view the pH value as an appropriate way to help students understand logarithms and the relationship between two quantities. In other words, Park and Choi state that the scientific meaning and the mathematical meaning of logarithms are deeply related. Therefore, educators need to establish a common teaching approach to logarithms to better ensure that students are conceptually understanding the mathematics and transferring their knowledge to the science contexts. A possible common

teaching approach to logarithms is a covariational approach (Confrey & Smith, 1995, Ferrari-Escolá et al., 2016).

In a covariational approach, teachers define logarithms “as the corresponding number in the arithmetic sequence” (Smith & Confrey, 1994, p. 340). Therefore, in mathematics, the logarithm is a relationship between a geometric and an arithmetic sequence and each property in an arithmetic sequence corresponds to a property of a geometric sequence (Confrey & Smith, 1995). Science teachers could transfer this approach to science by allowing students to relate the geometric sequence of hydrogen ion concentrations (powers of 10) to the arithmetic sequence of the pH scale using a double number line (Figure 2). In addition, students can establish corresponding properties between each sequence by investigating the behavior of pH as hydrogen ion concentration changes. For example, students can make observations about the double number line such as increasing pH by one will reduce the concentration of hydrogen ions by a factor of 10. From this observation, students can build the idea of relative differences in concentration based on pH values, such as a pH of 3 is one hundred times more concentrated than a pH of 5.

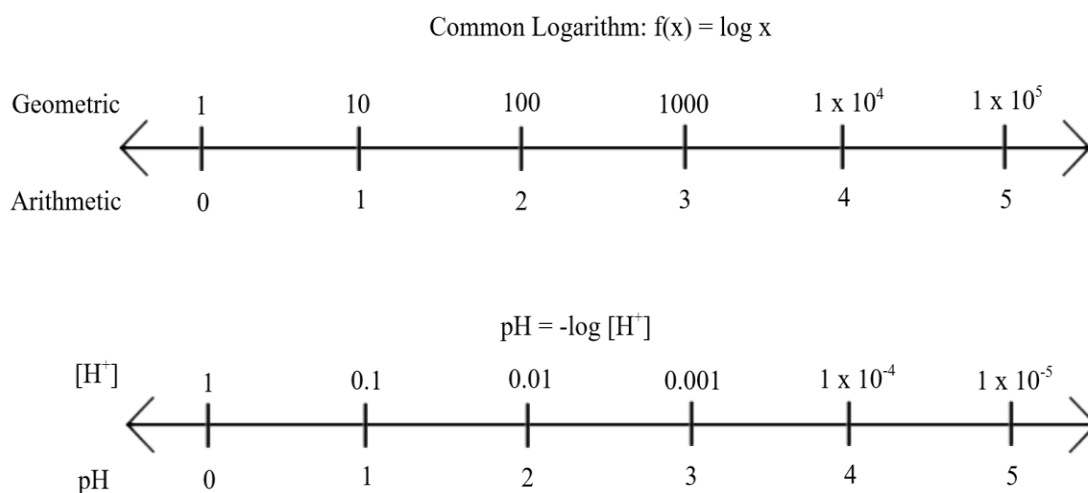


Figure 2: The common teaching approach: Covariational reasoning and the double number line

Using this common teaching approach, students can also transfer their knowledge of logarithms and pH to evaluate and estimate with logarithms in a similar manner without using a calculator. Leopold and Edgar (2008) stated when students use a calculator they can mask their deficiency in their conceptual understanding. Therefore, students that are given the opportunity of evaluating and estimating logarithms without a calculator could build their conceptual understanding by applying properties of logarithms or pH. For instance, the common logarithm is used in mathematics and to evaluate pH. If students understand that pH increases as the hydrogen ion concentration decreases and the square root of 10 is approximately 3.16, then students can estimate pH within 0.5 of the actual value similarly to how they would estimate values of the common logarithm. For example, to evaluate the common logarithm $\log(4.3 \times 10^4)$ and to evaluate pH when the hydrogen ion concentration is 4.3×10^{-4} (i.e. $-\log 4.3 \times 10^{-4}$), students can use the double number line (Figure 3). To evaluate these logarithms, students first must understand that the geometric sequence is a multiplying system and can be broken down into a smaller multiplying system using square roots. Therefore, the common logarithm is a base 10 system that can be broken down into approximately a 3.16 multiplying system. Once students have broken this system down, then they can fill in the missing values on the top and bottom of the double number line such as 4.5 and 3.16×10^4 (i.e. $\log 10^{4.5} \approx 3.16 \times 10^4$). As a result, students should be able to estimate the common logarithm within 0.5 of the actual value. Students can use a similar process to evaluate pH within 0.5. However, they would have to understand the indirect relationship between pH and hydrogen ion concentration. (The covariational approach and the double number line also build a foundation for reading logarithmic scales.)

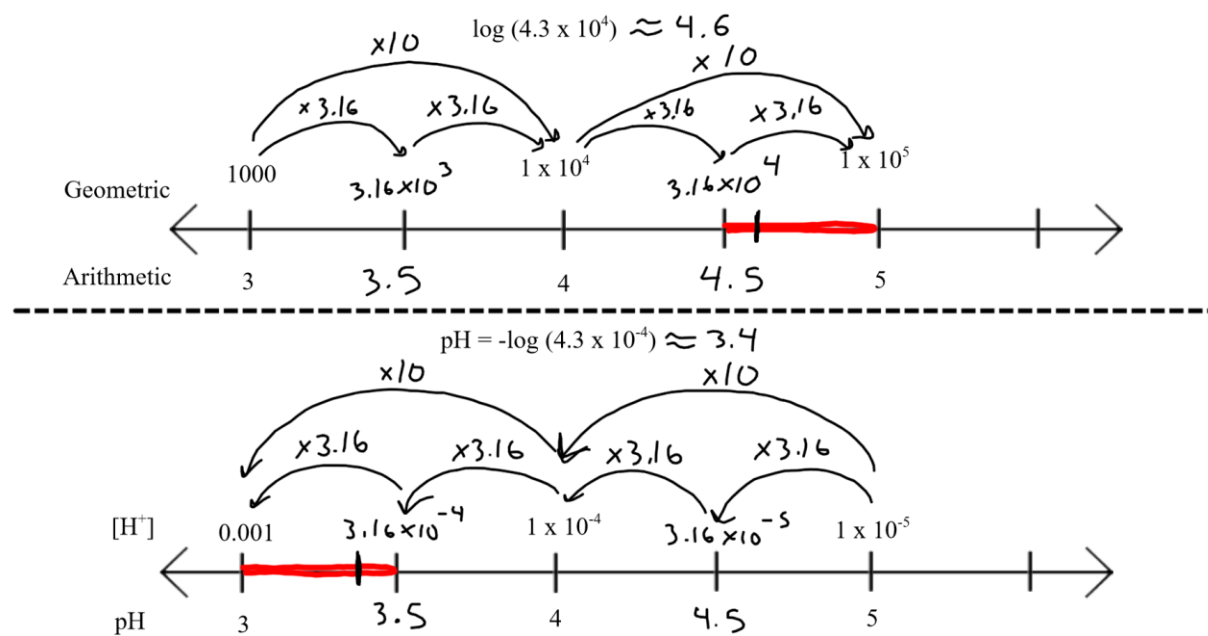


Figure 3: Estimating logarithms using covariational reasoning and the double number line

Chapter 3 Methodology

Purpose

The purpose of this research study was to examine the differences among students participating in a traditional classroom setting using a correspondence approach and students participating in an integrated STEM unit using a covariational approach. The researcher investigated differences in conceptual understanding of logarithms, in ability to make connections among multiple representations of logarithms, and in transfer of knowledge between mathematics and chemistry. The integrated STEM unit connected the concepts of logarithms from Algebra II and pH from chemistry, while the traditional classroom setting focused strictly on logarithms in mathematics and strictly on pH in chemistry. In other words, students participating in an integrated STEM unit were explicitly taught logarithms and pH in both mathematics and chemistry classrooms while students participating in a traditional classroom setting were implicitly taught logarithms in science and implicitly taught pH in mathematics.

Research Questions

This study investigated the following research questions:

1. How do lessons designed to teach logarithms conceptually using a covariational approach compare to traditional lessons using a correspondence approach in developing students' conceptual understanding of logarithms based on Weber's (2002) stages of conceptual understanding?
2. How do students using a covariational approach differ from students using a correspondence approach in making connections among different mathematical representations of logarithms?

3. How does a covariational approach to teaching logarithms and pH help develop a student's conceptual understanding of pH based on Park and Choi's (2012) domains of understanding pH?
4. How do students participating in the integrated STEM unit compare to students participating in the non-integrated unit on the Logarithms and pH Assessment?

Definition of Key Terms

The variables of this study have multiple definitions and interpretations. Therefore, definitions of key terms are provided.

Conceptual understanding. Conceptual understanding is defined as flexible knowledge that can be applied to multiple types of problems within a context (Panasuk, 2010; Rittle-Johnson, Siegler, & Alibali, 2001) such as representing logarithms numerically, algebraically, and graphically.

Covariational approach. A covariational approach is a teaching approach that views a function as the juxtaposition of two sequences, and the focus of the teaching approach is the corresponding properties between the two sets (Confrey & Smith, 1995). For example, exponential and logarithmic functions can be viewed as the juxtaposition of an arithmetic sequence (exponents) and a geometric sequence (powers)

Correspondence approach. A correspondence approach is a teaching approach that focuses and defines a function as the relationship between two sets (Confrey & Smith, 1995). The explicit formula for the relationship is the major focus.

Integrated STEM unit. An integrated STEM unit is a unit that uses a total integration approach that explicitly teaches topics from at least two STEM disciplines within each STEM classroom involved (Hurley, 2001) using a common teaching strategy. In this study, logarithms

were explicitly taught in Chemistry while being learned in Algebra II, and pH was explicitly taught in Algebra II while being learned in Chemistry by using a covariational approach.

Traditional unit. A traditional unit is a unit that does not explicitly integrate STEM disciplines. In this study, students participating in the traditional unit were taught logarithms in mathematics and pH in chemistry in parallel, but no connection between logarithms and pH were explicitly taught.

Written representation. Written representation refers to a student's ability to use vocabulary to define and explain concepts (Panasuk, 2010)

Numerical representation. Numerical representation refers to the students' ability to compare, evaluate, and estimate known quantities. (Lyons & Beilock, 2011).

Algebraic representation. Algebraic representation refers to the students' ability to use symbolic symbols and manipulate those symbols (Potgieter et al., 2008)

Graphical representation. Graphical representation refers to the students' ability to represent logarithms on a number line or coordinate plane and interpreting logarithms on a number line or coordinate plane (Potgieter et al., 2008).

Transfer. Transfer is the student's ability to apply his or her knowledge and skills from one context to another (Honey et al., 2014).

Research Design

This mixed-methods study utilized a quasi-experimental design by using existing groups of students and assigning those groups to either the treatment or control group. The students were divided into a control and treatment group based on the periods in which they were enrolled in Algebra II and Chemistry. The similarities between the control and treatment groups were compared by using the Georgia Algebra I End-of-Course assessment grades.

The study investigated students' conceptual understanding of logarithms and pH and the connections that students make between multiple representations of logarithms. In addition, the study investigated students' transfer of knowledge from one context to another. The Observation Protocol (Appendix B) and student work was collected to assess students' levels of conceptual understanding of logarithms based on Weber's (2002) stages of conceptual understanding of exponential and logarithmic functions. In addition, the Observation Protocol and student work were used to investigate students' transfer of knowledge between logarithms and pH as well as their conceptual understanding of pH based on Park and Choi's (2012) domains of understanding pH. The Logarithms and pH Assessment instrument (LPA) (Appendix C) was used as a quantitative measure to compare students' levels of conceptual understanding of logarithms and pH. The LPA compared the students' ability to represent and use logarithms and pH in multiple ways. The LPA was designed after Park and Choi's (2012) interview protocol, logarithmic questions from the SAT, and logarithmic questions from the Georgia Standards of Excellence frameworks and tested for validity and reliability during a pilot study.

Integrated STEM Lessons

The integrated STEM unit combines logarithms from Unit 4 from Algebra II (Georgia Standards of Excellence, 2015) and pH from the Acids and Bases unit from GSE Chemistry. The logarithms unit for Algebra II followed the progression of developing the definition of logarithms suggested by Hamdan (2008) and incorporated a covariational approach used by Ferrari-Escolá et al. (2016) to develop a student's conceptual understanding of logarithms. The pH unit for Chemistry focused on developing a student's conceptual understanding of pH by building a foundational knowledge based on the mathematical relationship between the hydrogen ion concentration and pH.

Logarithms Unit. According to Hamdan (2008), students should first develop an awareness of exponents with integer powers, and then second, investigate the idea that an exponential function is continuous. Students can develop their knowledge of integer powers and continuity by investigating the relationship between rates of change of an arithmetic sequence (domain) and a geometric sequence (range) of exponential functions (Ferrari-Escolá et al., 2016). In other words, students can develop their knowledge of exponential and logarithmic functions by using a covariational approach. Therefore, students started by using the Base Two Card Game Task (Appendix D) to investigate the powers of two (2^n) using covariational reasoning and plotting x versus 2^x on the coordinate plane to identify key features of the graph. Then, students were given the opportunity to develop an understanding of the continuity of exponential functions by using covariational reasoning to understand the half power. In addition, students began to investigate rates of change of x versus rate of change of 2^x (i.e. $n + m$ is related to $2^n \cdot 2^m$) to help develop their understanding of the properties of exponents, and they began to investigate the inverse relationship of the exponential function. However, the notation of the inverse function was not discussed.

After students investigated the idea of a continuous exponential function, students explored properties of exponential functions (Hamdan, 2008). Though Hamdan does not introduce the idea of logarithms until after students have developed their understanding of exponential functions, the properties of logarithmic functions can be developed along with the properties of exponential functions through covariational reasoning (Ferrari- Escolá et al., 2016). Students continued to explore exponential and logarithmic functions by investigating the relationship between rates of change of the domain of the exponential function (range of the logarithmic function) and the rates of change of the range of the exponential function (domain of the

logarithmic function) for a variety of bases (2, 3, 4, and 10). Through the Different Bases task (Appendix E), students investigated the power of a product, the power of a quotient, the power of a power, the negative exponent, and the zero exponent properties and the corresponding logarithmic properties by applying them to different bases using a covariational approach. Students investigated graphs of both exponential and logarithmic functions and identified key characteristics of the graphs. The logarithmic notation was introduced at the end of the task and students were expected to write relationships in both an exponential and logarithmic form.

Hamdan (2008) stated that “the development of a concept is naturally spiral in the sense that it gets more complete and mature as more information and richer mathematical awareness is reached at different stages” (p. 546). Once the first two tasks are complete, students have been introduced to all the required knowledge to complete the integrated STEM unit. Therefore, the remaining tasks and lessons reinforced the intended knowledge gained, allowed students to use their knowledge of logarithms to investigate pH, and allowed students to apply their knowledge to multiple representations (written, numerical, algebraic, and graphical).

The pH and Logarithms Task (Appendix F) allowed students to use their mathematical knowledge of logarithms and covariational reasoning to investigate the relationship between the pH value and hydrogen ion concentration. Students investigated rates of change to develop the understanding that as pH increases the hydrogen ion concentration decreases. Students also investigated proportional differences between hydrogen ion concentrations for different pH values. In addition, they were given the opportunity to make connections to the Cabbage Juice Lab from the pH unit and the pH of common household products.

During lessons four and five, students were involved in developing their numerical understanding of logarithms by evaluating, comparing, and estimating logarithms, and

developing their algebraic representation by applying properties of logarithms learned in previous tasks. Students were not allowed to use calculators during this lesson to help students develop their conceptual understanding (Leopold and Edgar, 2008). pH was incorporated into these lessons during the pH scale activity, in which students estimated pH values for known hydrogen ion concentrations and estimated hydrogen ion concentrations for known pH values. The pH scale activity also built on the foundational knowledge for developing the graphical understanding of logarithms which began in lessons one through three.

Lesson six focused on a student's ability to use logarithms algebraically. Students were asked to solve equations with rational solutions without a calculator, and students were asked to estimate the solution to a logarithmic equation with irrational solutions. In addition, students were asked to find pH and hydrogen ion concentrations for unknown concentrations and pH values. The focus of the lesson was on covariational reasoning instead of algebraic manipulation. For example, to solve $\log_2(x + 5) = 3$, students with developed covariational reasoning understood that three is equivalent to eight in a base two system, and therefore understood that $x + 5 = 8$ without converting the logarithmic equation to an exponential form.

Lesson seven focuses on a student's ability to understand logarithms graphically. Students were given equations, asked to make a table, and graph the function. The equation included parent functions ($f(x) = \log_b x$) and functions that involve transformation of the parent functions. In addition, students were asked to identify characteristics of the functions such as domain, range, intercepts, asymptotes, and rates of change (increasing or decreasing). Students also asked to analyze the pH function ($pH = -\log[H^+]$) and identify key features of the graph in terms of chemistry.

In summary, students were asked to use a covariational approach and given the opportunity to represent logarithms in multiple forms with the intent of developing their conceptual understanding. They were asked to evaluate, compare, and estimate to gain a numerical understanding of logarithms. They were asked to use properties and solve equations to help develop their algebraic understanding. They were asked to make tables, graphs, and identify key features to develop their graphical understanding of logarithms. In addition, students were asked to write and verbalize their understanding of logarithms while using multiple representations and applying their knowledge to science by using numbers, formulas, and graphs to explain pH in the chemistry context.

pH Unit. According to Park and Choi (2012), numbers, formulas, and graphs are an important part in understanding scientific phenomena, because they help to describe and explain. Therefore, the goal of the Chemistry unit was to expose students to the scientific phenomena of pH through hands-on experiences and use numbers, formulas, and graphs to help students develop their conceptual understanding of pH.

In lesson one, students were introduced to pH by participating in a cabbage juice lab. Students used cabbage juice to identify the pH of common household products and order them from least to greatest in terms of pH. The goal of the lab was to show that there is a molecular difference in the substances that causes the cabbage juice color to change. The Algebra II teacher used this molecular difference to help students understand that logarithms can help explain extremely small differences in concentrations.

In lessons two and three, students were introduced to common terminology used with and characteristics of acids and bases with an emphasis on Arrhenius acids and bases, which are compounds that disassociate in water to donate hydrogen or hydroxide ions in solution (Georgia

Standards of Excellence, 2015). They then proceeded to use these terms within the chemistry and algebra II classrooms. The purpose of the lesson was to allow students to gain knowledge of and use scientific terminology to better understand pH. For example, when students are asked to analyze the pH function, they should be able to use terminology from both science and mathematics. In addition, the mathematics teacher used these terms within the mathematics classroom to help students make a connection between mathematics and science concepts.

In lesson four, students used an online interactive simulation of the molecular interactions of acids and bases. The simulation allowed students to see the molecular structure and formula of the dissociation of the Arrhenius acid and base. In addition, students analyzed the numerical values of the concentration of hydrogen ions on a bar graph to compare and contrast weak and strong acids and bases. The goal of the lesson was for students to understand the meaning of a weak and strong acid and base by analyzing the molecular and numerical representations.

In lesson five, students used the negative logarithm of the hydrogen ion concentration or hydroxide ion concentration to calculate various pH and pOH values, respectively. In addition, students were introduced to the ionization constant of water, and used this constant to calculate pH and pOH. The goal of the lesson was to allow students to connect their mathematical knowledge of logarithms to science. Students should be able to calculate pH and hydrogen ion concentration for known concentrations and pH values. In addition, students investigated the connection between the asymptote of the pH function and the ionization constant of water to help explain why the concentration of hydrogen ions cannot be zero.

In lesson six, students were required to research a commercially produced acid and base. They discovered its uses and described its characteristics using terminology from both math and

science. The goal of the lesson was to allow students to use terminology, gain insight into the uses of acids and bases, and help students gain a deeper understanding of pH.

In summary, the pH unit provided students with hands-on experiences, academic language, and mathematical descriptions of pH. Students participated in labs, class activities, and projects to help develop the knowledge necessary to conceptually understand pH. In addition, a major focus of the pH unit was using mathematics as a way to describe the characteristics of a scientific topic. Therefore, the goal of the integrated STEM unit was to help students make connections between mathematics and science by explicitly teaching logarithms and pH within each classroom setting.

Traditional Logarithms and pH units

The traditional logarithms and pH units focused on logarithms and pH, but the topics were not explicitly taught as connected during instruction. Students only received explicit instruction on logarithms in the mathematics classroom, and students only received explicit instruction on pH in the chemistry classroom. Though students saw logarithms in chemistry and pH in mathematics, they only used those topics to serve the purpose of the class. For example, students used pH in mathematics only as a setting in which to evaluate logarithmic expressions or solve logarithmic equation. In chemistry, students used logarithms only to define and evaluate pH given a hydroxide ion concentration.

Traditional logarithms unit. Like the integrated logarithms unit, the traditional logarithms unit for Algebra II followed the progression of developing the definition of logarithms suggested by Hamdan (2008), but incorporated a correspondence approach, which relates the domain to the range by an explicit rule (Confrey & Smith, 1995). The focus of the unit was on algebraic

manipulations and procedural skills, and students were expected to view logarithms as the inverse of an exponential function (Georgia Standards of Excellence, 2015).

In lesson one, students participated in the “Investigating Exponential Growth and Decay” task designed by Georgia Department of Education (Georgia Standards of Excellence, 2015). Students investigated exponential functions in which the domain is the number of times a piece of paper is folded in half and the range is the number of sections or the area of each section. Students explored integer powers, which is the main goal of the lesson.

In lesson two, students participated in the “Graphs of Exponential Functions” task designed by Georgia Department of Education (Georgia Standards of Excellence, 2015). Students graphed basic exponential functions ($y = b^x$) and investigated transformations of those functions. Students explored the idea that an exponential function is a continuously increasing or decreasing function and is one-to-one. In addition, they were asked to determine key characteristics of exponential functions such as domain, range, asymptotes, and end-behavior.

In lesson three, students participated in the “Bacteria in the Swimming Pool” task designed by Georgia Department of Education (Georgia Standards of Excellence, 2015). Students were involved in developing a table for and analyzing the structure of an exponential function that involves the growth of bacteria. The students calculated the amount of bacteria present at a certain time and then tried to calculate the time needed to reach a certain level of bacteria. The task led students to understand that the inverse of an exponential function is necessary to solve an exponential equation. However, the notation for the function was not introduced until the next lesson.

In lesson four, students participated in the “What is a Logarithm?” task (Appendix G) designed by Georgia Department of Education (Georgia Standards of Excellence, 2015). The

task introduced students to the logarithmic notation and presented logarithms as the inverse of an exponential function. The task focused on converting exponential equations to logarithmic equations, solving exponential equations by converting to the logarithmic form, and evaluating and estimating logarithms.

The remaining lessons were the same as the integrated STEM unit, but pH was excluded from the lessons. Students were involved in estimating, evaluating, and comparing logarithmic expressions, using properties of logarithms to solve equations, and graphing and identifying key features of logarithmic functions. Students were taught procedural skills necessary for calculating pH in chemistry, but they did not receive any direct instruction of how this was done.

In summary, the traditional logarithms unit involved students in developing their understanding of logarithms, but it was not focused on developing a student's ability to connect the concept to pH. The unit followed Hamdan's (2008) progression of developing the definition of logarithms but focused on a correspondence approach that is mainly taught in a traditional classroom (Confrey & Smith, 1995). The lessons at the beginning of the unit were different from the integrated STEM unit because of the correspondence approach to teaching logarithms. However, the other lessons were identical in content but different in teaching approach and integration of pH.

Traditional Chemistry unit. The traditional chemistry unit varied from the integrated unit, because the traditional unit did not explicitly use mathematics to explain pH. The lessons covered the same topics at the same pace, but the students were not explicitly taught how to use logarithms to evaluate pH or find a hydrogen ion concentration. Instead, students relied on a scientific calculator to perform calculations and were directly told through lecture the relationship between the hydrogen ion concentration and the pH values. The chemistry teacher

did not use mathematics as a way to help explain the relationship between pH and the hydrogen ion concentration. Students were expected to transfer that knowledge from mathematics to science. Though students used logarithms in the chemistry classroom, logarithms were used only to calculate pH and not as a way to understand the characteristics of pH.

Pilot Study

The purpose of the pilot study was to design and refine the lessons for the integrated STEM unit and design and establish a valid and reliable assessment tool to measure the conceptual understanding of logarithms and pH. Teacher participants in the study included two high school certified mathematics teachers, two high school pre-service teachers, and a high school certified science teacher. Student participants included 55 on-level Algebra II students, 26 Algebra II honor students, and 8 calculus students. Only 40 of the on-level Algebra II students and 8 of the Algebra II honor students were also enrolled in chemistry. The pilot study took place during the second semester of the 2016-2017 school year at a rural high school in northwest Georgia. The duration of the pilot study was four weeks.

The lessons were designed, implemented, and reviewed by each participating teacher. During the implementation of a lesson, teachers would monitor the students and make notes based on observations and questioning. Then after each lesson, the teachers would review the notes and any improvements that could be made to the lessons were incorporated. In addition, before the lessons were implemented to the Algebra II students, the eight calculus students were given the opportunity to participate in the first three lessons of the integrated logarithms unit. Because the calculus students have learned logarithms, they were given the opportunity to provide feedback to improve the lessons through think-aloud sessions. Improvements to the lessons included the elimination of needless repetition, rewording of questions and instructions,

and improvements to instructional strategies. For example, in the Base Two Card Game task, students were initially becoming confused by the manipulatives. Therefore, instructions were rewritten, and implementation strategies were discussed among the teachers to improve the flow of the lesson.

The LPA instrument was designed prior to the implementation of the integrated STEM unit. The questions for the assessment were modeled after the Park and Choi (2012) interview protocol, the Leopold and Edgar (2008) study on mathematical fluency and success in chemistry, the Heckler et al. (2013) study on students' ability to read logarithmic plots, and the GSE standards (2015). Each participating teacher reviewed the instrument before the initial implementation to establish content validity. After the implementation of the instrument, students were chosen, based on the similarities in answers, to participate in a "think-aloud" session to determine if the students were using the intended representation to solve the problem or if students' mistakes were based on a lack of understanding of the concepts or based on the structure of the instrument. In addition, a factor analysis was conducted on both the mathematics and science portion of the assessment, which was used to eliminate or reword questions.

Rubrics were created by the participating teachers for each open-ended response question based on the Park and Choi's (2012) study and student responses to the LPA instrument. After the initial rubrics were created, 15 randomly selected tests were graded by at least three participating teachers and compared to establish interrater reliability. After the firsts grading cycle and discussion, another 15 randomly selected tests were chosen and graded independently. The high school certified science teacher only participated in making the rubrics for the pH open-ended questions.

Development of Logarithms and pH Instrument

For this study, conceptual understanding in Algebra is the ability of a student to recognize functional relationships and to interpret different representations of the concept (Panasuk, 2010). Students can express their conceptual understanding through verbal, visual, and symbolic representations, and the flexible use of these representations on a certain concept is associated with a deep level of conceptual understanding. Therefore, to quantitatively measure conceptual understanding, the Logarithms and pH assessment (LPA) was designed to measure four representations of logarithms and pH (written, numerical, algebraic, and graphical) in both Algebra II and Chemistry.

The LPA contained 32 questions about logarithms that cover the related Georgia Standards of Excellence (GSE) standards in both eleventh grade Algebra II and Chemistry. The focus of each question was on logarithms in a mathematics context or logarithms in a chemistry context, because the study was looking at the conceptual understanding of logarithms in a mathematics and science classroom. In addition, conceptual understanding was defined as flexible knowledge that can be applied to multiple types of problems within a context (Rittle-Johnson et al., 2001) and knowledge that can be transferred to multiple settings. Therefore, the mathematics and science contexts contained 16 questions each, and within each context, four questions of each type (written, numerical, algebraic, and graphical) were developed. All questions are coded with either M for mathematics or S for science, and W for written, N for numerical, A for algebraic, or G for graphical. For example, a question coded MG3 is the third graphical question in the mathematics section.

Creation of rubrics. To measure the level of conceptual understanding expressed by the students, rubrics were designed for each open-ended question and tested during the pilot study.

The rubrics for logarithms in a mathematics context were used by four secondary mathematics teachers on 15 randomly selected tests and the rubrics for logarithms in a science context were used by one secondary science teacher and two mathematics teachers who participated in the design and implementation of the integrated STEM lessons. The teachers graded the questions independently and then discussed any differences until a consensus was reached and rubrics were adjusted. Then the cycle was repeated with the adjusted rubrics and with 15 more randomly selected tests. All revised rubrics had a percent agreement of 80% or more.

Final Logarithms and pH Assessment Instrument

After the completion of the Logarithms and pH units and the administration of the LPA, a factor analysis was conducted on the 16 mathematics content questions and the 16 science content questions. Items that did not load well on a factor were eliminated and factors that contained questions that appeared to be unrelated were not considered because the correlation was due to incorrect responses and not related knowledge.

The 16 mathematics content items were subject to a principle components analysis (PCA) using SPSS Version 25. Prior to the PCA, the suitability of using a factor analysis was conducted. The correlation matrix revealed the presence of many coefficients of .3 and above. In addition, the Kaiser-Meyer-Olkin value of 0.829 (>0.6) and the statistical significance of the Bartlett's Test of Sphericity indicated the appropriateness of the factorability of the 16 mathematics content items.

The PCA revealed the presence of three components with Eigenvalues above one (Table 4), explaining 35 percent, 10 percent, and 8 percent of the variance respectively. Upon further examination of the test, the components that explained 10 and 8 percent of the variance did not follow any logical component. The correlation among the items seemed to be incorrect

responses and not a relationship among the items that could be explained by a student's conceptual understanding of logarithms. Therefore, these two components were not considered and the items (MW1, MN4, MA1, MA2, MA3, MA4, and MG1) that loaded strongly on these components were eliminated from the overall score on the assessment. After the initial PCA, a reliability analysis (Table 5) was conducted on the remaining nine items (MW2, MW3, MW4, MN1, MN2, MN3, MG2, MG3, MG4) and returned a Cronbach's alpha of 0.835 (>0.70). In addition, the written representation items of the component (MW2, MW3, MW4) returned a Cronbach's alpha of 0.771 (>0.70), and the numerical representation items of the component (MN1, MN2, and MN3) had a Cronbach's alpha of 0.653. Therefore, these groups of items were used to further compare groups. However, the graphical representation items (MG2, MG3, MG4) returned a Cronbach's alpha of 0.548. Therefore, this group of items was not used to further compare groups

Table 4

Total Variance Explained (Mathematics Context Questions)

<u>Component</u>	<u>Total</u>	<u>Initial Eigenvalues</u>	
		<u>% of Variance</u>	<u>Cumulative %</u>
1	5.641	35.256	35.256
2	1.607	10.043	45.299
3	1.239	7.742	53.041

Table 5

Reliability Analysis (Mathematics Context Questions)

<u>Items</u>	<u>Cronbach's Alpha</u>	<u>N of Items</u>
MW2, MW3, MW4, MN1, MN2, MN3, MG2, MG3, MG4	.835	9
MW2, MW3, MW4	.771	3
MN1, MN2, MN3	.653	3
MG2, MG3, MG4	.548	3

The component that was extracted from the PCA contains items that require students to communicate their conceptual understanding of logarithms through writing, numerical representations, and graphical representations. Because students are required to interpret logarithms in multiple representations and conceptual understanding for this study was defined as flexible knowledge that can be applied to multiple types of problems within a context (Panasuk, 2010; Rittle-Johnson et al., 2001), this component was defined as conceptual understanding of logarithms in a mathematics context (CU). In addition, the CU component was split into two sub-components, conceptual understanding represented through written representations (CW) and conceptual understanding represented through numerical representation (CN).

The 16 science content items were subject to a PCA using SPSS Version 25. Prior to the PCA, the suitability of using a factor analysis was conducted. The correlation matrix revealed the presence of many coefficients of .3 and above. In addition, the Kaiser-Meyer-Olkin value of 0.805 (>0.60) and the statistical significance of the Barlett's Test of Sphericity indicated the appropriateness of the factorability of the 16 science content items.

The PCA on the science content items revealed the presence of five components with Eigenvalues above one (Table 6), explaining 31.7, 9.3, 7.5, 7.0, and 6.6 percent of the variance respectively. Upon further examination of the test, the components that explained 31.7, 7.5, and 7.0 of the variance did not follow any logical component. The correlation among the items seemed to be incorrect responses and not a relationship among the items that could be explained by a student's conceptual understanding of pH. Therefore, component one, three, and four were not considered and the items (SW1, SN1, SN2, SN3, SA2, SA3, SA4, SG2) that loaded strongly on these components were eliminated from the overall score on the assessment. After the initial PCA, a reliability analysis (Table 7) was conducted on components two and five. The component containing SN4, SG1, SG4, and SA1 had a Cronbach's Alpha of 0.689 and the component containing SW2, SW3, SW4, and SG3 had a Cronbach's Alpha of 0.775. In addition, the components were combined to create an overall science component, which had a Cronbach's Alpha of 0.775.

Table 6

<i>Total Variance Explained (Science context questions)</i>			
<u>Component</u>	<u>Initial Eigenvalues</u>		
	<u>Total</u>	<u>% of Variance</u>	<u>Cumulative %</u>
1	5.070	31.690	31.690
2	1.486	9.290	40.980
3	1.207	7.542	48.522
4	1.135	7.094	55.616
5	1.050	6.562	62.178

Table 7

Reliability Analysis (Science Context Questions)

<u>Items</u>	<u>Cronbach's Alpha</u>	<u>N of Items</u>
SW2, SW3, SW4, SG3	.775	4
SN4, SG1, SG4, SA1	.689	4
SW2, SW3, SW4, SG3, SN4, SG1, SG4, SA1	.775	8

The components that were extracted from the PCA contain items that required students to evaluate pH values given the hydrogen ion concentration and to express their conceptual understanding of the logarithmic relationship between pH and hydrogen ion concentration through writing. The component containing SN4, SG1, SG4 and SA1 required students to evaluate pH given a hydrogen ion concentration. In SG1 and SG4, students are provided the pH scale (logarithmic scale) and can use covariational reasoning to solve the problem. On SN4 and SA1, students can draw the pH scale and use covariational reasoning to solve the problem. Therefore, this component measured a student's ability to estimate and evaluate pH using logarithms and was defined as the "numerical representation in a science context" or SN. The second component required students to express their understanding of pH through writing. Students must use the logarithmic definition of pH to interpret the meaning of a mathematical expression in the context of science, and students must express their understanding of the range of both pH and hydrogen ion concentration. Because students were required to represent their understanding through writing, the second component was defined as "written representation in a science context" or SW. To measure a student's overall performance on the science context questions, the scores from the SN and SW components were combined to form the SO component.

The final instrument measured a student's conceptual understanding of logarithms by providing a score for conceptual understanding in mathematics and in science through written communication and numerical representation (Figure 4). The open-ended questions measuring the written representation were graded using a rubric designed using Park and Choi's (2012) levels of understanding of pH and Weber's (2002) levels of understanding exponential and logarithmic functions

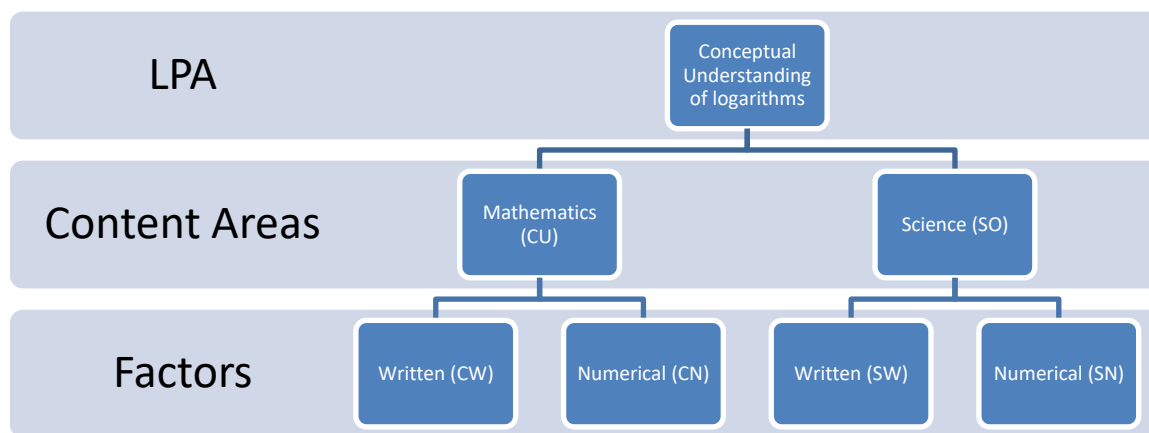


Figure 4: LPA structure

Participants

Participants in this study included 40 high school juniors located at a rural title-one school in the Southeastern United States that were enrolled in on-level Algebra II and Chemistry courses in the fall semester of the 2017-2018 school year. The school population is approximately 860 students consisting of 80% White with more than 60% who qualify for free or reduced lunch. Participation in the study was based on the completion and return of a parent consent form and student assent form. Once consent was established, control and treatment groups were created based on the students' class schedule. The control group consisted of 17 students (13 White, 3 Hispanic, 1 bi-racial) and the treatment group consisted of 23 (17 White, 4

Black, 2 Hispanic) students. The groups were determined to be statistically similar by comparing their Algebra I EOC test scores ($t = -0.400$, $df=38$, $p = 0.691$) collected from student transcripts.

The teachers conducting the lessons were determined based on the subjects they taught. The mathematics teacher, who was also the researcher, was the only teacher in the school with on-level Algebra II courses and the science teacher was the only teacher in the school with on-level chemistry courses. In addition, both teachers participated in a pilot study in which they designed the integrated STEM unit and collaborated on ways to improve its implementation. Therefore, both educators have experience designing and implementing an integrated STEM unit on logarithms and pH. The experience is important to the study because the researcher has determined that a properly integrated STEM unit will constantly involve collaboration among teachers from different content areas in the planning and implementation process.

Procedure

The researcher received IRB approval (Appendix H) from Kennesaw State University and from the local board of education after submitting the appropriate documentation, research instruments, and consent forms. Then, the researcher, a high school mathematics teacher, and a Chemistry teacher implemented the integrated STEM lessons relating logarithms and pH to the treatment group and implemented traditional lessons on logarithms and pH based on the state frameworks to the control group during the fall semester of the 2017-2018 school year. The pacing of both the traditional and integrated lessons were the same, and the start of the pH unit in both the control and treatment group was delayed two days (Appendix I), because pH is a smaller part of the chemistry curriculum than logarithms are of the algebra II curriculum. Teachers in mathematics require more time to cover the GSE standards on logarithms than

chemistry teachers need to cover the standards related to pH (Georgia Standards of Excellence, 2015). In addition, the two-day delay allowed for a better alignment of the lessons during the integrated STEM unit. Therefore, the activities were designed to allow students to apply their knowledge from both chemistry and Algebra II in the other content area within a shorter time.

The integrated STEM lessons were developed during the pilot study through collaboration between both teachers. The unit lasted approximately three weeks and ended with all participating students taking the Logarithms and pH assessment instrument. During each lesson in mathematics and science, both teachers used the Observation Protocol to make observations based on Weber's (2002) levels of conceptual understanding of exponents and logarithms and Park and Choi's (2012) domains of conceptual understanding of pH. When collecting and reporting the qualitative data, pseudonyms were used to eliminate any identifying information. In addition, the logarithms part of the assessment instrument and the pH part of the assessment instrument were given on consecutive days at the end of the integrated STEM and traditional units. The mathematics teacher administered the logarithms assessment and the chemistry teacher administered the pH assessment. Then the researcher analyzed and tested the components of the LPA for statistical significance.

Because the researcher was the mathematics instructor, the logarithmic units, data collection, and data analysis were designed to limit bias. First, the units were part of a collaborative effort between the mathematics and chemistry instructors. Both units were based on Hamdan's (2008) progression of developing the definition of logarithms, and the lessons covered the same concepts at the same pace. The only difference in the units was the intended difference investigated in the study. In the traditional unit, the mathematics instructor used a correspondence approach to teaching logarithms, and the integration of logarithms and pH was

omitted from the lessons. In the integrated STEM unit, the mathematics instructor used a covariational approach and integration of pH was made explicit in mathematics. In addition, the lessons in both the integrated STEM and traditional units were designed to allow students to gain the knowledge to answer each question on the LPA. At the end of each lesson, the mathematics instructor would conduct a whole class discussion to review the intended goals of the lesson. Second, the mathematics instructor collected observational data and student work in both the traditional and integrated STEM units. The mathematics instructor would collect any observational data that showed different levels of conceptual understanding of logarithms according to Weber (2002), different levels of conceptual understanding of pH according to Park and Choi (2012), and evidence of conceptual understanding through multiple representations (Panasuk, 2010). In addition, evidence of transfer was collected according to Honey et al.'s (2014) definition of transfer. Lastly, the mathematics instructor analyzed the observational data and student work by using codebooks (Appendix J) and the LPA data were scored by using rubrics that were collaboratively designed during the pilot study.

Data Analysis

Data from the Observation Protocol and student work were collected to measure levels of conceptual understanding expressed by students based on Weber's (2002) levels of conceptual understanding of exponents and logarithms, on Park and Choi's (2012) domains of conceptual understanding of pH, and on Panasuk's (2010) definition of conceptual understanding. In addition, data from the Observation Protocol and student work were collected to measure transfer of knowledge between logarithms and pH. Transfer is the student's ability to apply his or her knowledge and skills from one context to another (Honey et al., 2014). The qualitative data collected from the Observation Protocol and student work were coded using the Conceptual

Understanding of Logarithms, Conceptual Understanding of pH, and Transfer codebooks (Appendix J).

The researcher compared the qualitative data collected from student work and the observation protocol. The researcher used codebooks based on Weber's (2002) Stages of Understanding Exponents to assess students' understanding of logarithms in a mathematics context and Park and Choi's (2012) domains of conceptual understanding of the pH value to assess students' understanding of logarithms in a science context. In addition, a codebook was used to compare students in both the control and treatment groups on their ability to connect multiple representations and transfer their knowledge between logarithms and pH. The researcher also provided counts to illustrate the frequency in which student work showed certain levels of conceptual understanding. However, the level of understanding in each lesson is the level at which a student was observed performing the task with or without coaching and not necessarily the level that was retained. In addition, the differences in the correspondence approach and the covariational approach are highlighted.

Data from the Logarithms and pH assessment instrument were collected and analyzed to measure a student's conceptual understanding when defined as flexible knowledge that can be applied to multiple types of problems within a context (Panasuk, 2010; Rittle-Johnson et al., 2001). The control and treatment groups were compared in both logarithms in a mathematics context and logarithms in a chemistry context on the components found on the logarithms and pH assessment factor analysis. A student earned a point for a correct answer or zero points for an incorrect answer on a multiple-choice question. On the open-ended questions, student responses were graded using the rubrics (Appendix K) for each problem and given a score ranging from zero to four. The overall scores on the mathematics content questions and the science content

questions were measured by adding the component scores from the mathematics components and the component scores from the science components respectively. To determine if a statistically significant difference existed between the control and treatment group a Mann-Whitney U test (Appendix L) was performed on each component (CW, CN, CU, SN, SW and SO).

Chapter 4: Data Analysis and Findings

The purpose of this research study was to examine the difference in conceptual understanding of students participating in a traditional classroom setting and students participating in an integrated STEM unit. In this chapter, the researcher presents the findings from the study that investigated the impact an integrated STEM unit has on students' conceptual understanding of logarithms and pH. The researcher presents the findings by first analyzing qualitative data that illustrates the differences in conceptual understanding of the control and treatment groups. Following the qualitative results, the researcher presents the quantitative data to determine if any statistically significant difference in conceptual understanding as measured by the logarithms and pH assessment exist between the control and treatment groups.

Qualitative Results

In this section, the qualitative data are presented in order of the student work and observation protocol by lesson. The researcher analyzed lesson one through four separately for the control and treatment groups, because the lessons were different based on the correspondence and covariational approach as well as the STEM integration of pH. Only observations were made, and no student work was collected in lessons six, seven, and eight. Therefore, overall counts of student data are not available for those lessons.

Lesson 1. The control group participated in the “Investigating Exponential Growth and Decay” task (GSE, 2015), which required two class periods of 50 minutes. Students were expected to use a correspondence approach (Confrey & Smith, 1995) to represent exponential growth and decay functions using a table, an algebraic equation, and a graph. Students investigated the number of sections as a function of the number of folds ($f(x)=2^x$), and the area of a section compared to the whole paper as a function of the number of folds ($f(x) = (\frac{1}{2})^x$). The

treatment group participated in the “Base 2 Card” activity (Appendix D), which required three class periods of 50 minutes. Students were expected to use covariational reasoning (Ferrari-Escolá et al., 2016) to develop their knowledge of the relationship between the exponents and powers of two ($f(x)=2^x$). Therefore, during lesson 1, the control group focused on input and output of the function $f(x)=2^x$, and the treatment group focused on the rate of change of the function $f(x)=2^x$.

In the control, students were first asked to fill in a table that related two sequences, the number of folds to the number of sections and the area of each section (Figure 5). From this numerical representation, students were asked to explain if the relationship between the number of folds and the number of sections is a function. The most common answer given was an example of a covariational approach to understanding the relationship. For example, Cheer stated “for every one-fold, the sections double.” No student was observed, and no student work was turned in that indicated that they understood if the relationship was a function, but the students were aware of the patterns in the sequences. In addition, most students found it difficult to represent the numerical relationship algebraically. Only one student was observed making the connection that the relationship between the number of folds and the number of sections was powers of two. Kari stated that he noticed the number of sections were powers of two and therefore created the function $f(x) = 2^x$. However, the students were shown by the teacher the algebraic function that was represented by the table at the end of the task.

Number of Folds	0	1	2	3	4	5
Number of Sections	1	2	4	8	16	32
Area of each section compared to area of original paper	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

Figure 5. Lesson 1: Control Group's Table

After examining the created table, the control group was directed to make a graph of the table for each relationship and then describe the relationship as exponential growth or decay. With this graphical representation, the most common answer described the relationship as increasing or rapidly increasing. For example, Caitlynn (Figure 6) stated “it starts off low and gets higher and higher.” A similar result was observed when students graphed the relationship between the number of folds and the area of each section. Students described the relationship as decreasing. Only two students were observed extending this relationship beyond the five folds required by the task. Cory stated that “if it was possible to fold the paper more, the area would get smaller and smaller”. Mason reacted to this statement by saying that “you cannot fold the paper enough times to make it disappear”. Therefore, the highest observed level of understanding in the “Investigating Exponential Growth and Decay” task (GSE, 2015) was “exponentiation as a process” (Weber, 2002). Students were able to describe the functions $f(x) = 2^x$ and $f(x) = \left(\frac{1}{2}\right)^x$ as increasing or decreasing, and students were only able to describe this relationship using whole number exponents.

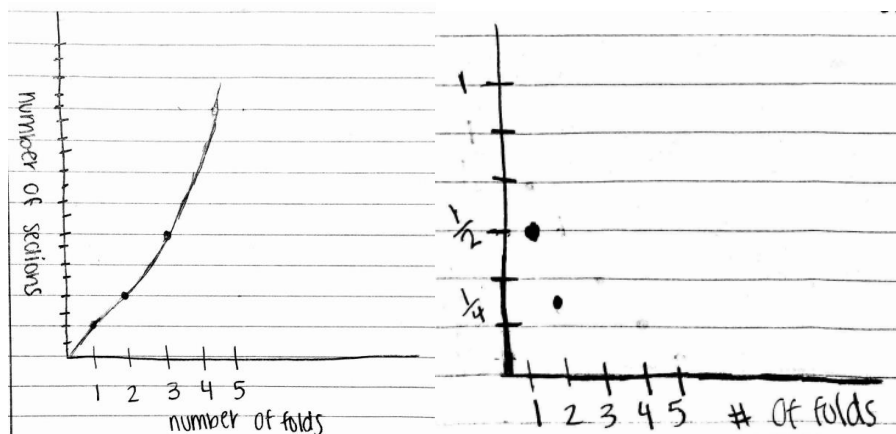


Figure 6. Student work: Caitlynn

In the treatment group, students were first asked to create more cards for a game in which only four cards were provided (Figure 7). But instead of focusing on the function that relates the two sequences together, students in the treatment group were asked to find rules (properties of logarithms and exponents) that govern the game. Through the numerical representation of the cards, the first rules that students were asked to discover were the product and quotient rules for logarithms and exponents. Students were observed discovering these rules by being directed to add the numbers on the bottom of the cards and observe the change in the top of the cards. For example, Jim used the cards $2 \rightarrow 4$ and $4 \rightarrow 16$ to find $6 \rightarrow 64$. 2 and 4 add to 6, and 4 and 16 multiply to 64. Therefore, he concluded that the rule for multiplication of these cards was to “multiply the top numbers and add the bottom numbers” (Figure 8). In addition, Jim was able to use his knowledge of the multiplication rule and develop the division rule. Therefore, in the first part of the task, the treatment group was able to reach an “Exponential expressions as the result of a process” level of understanding (Weber, 2002) by building the foundation of the product and quotient rules for exponents and logarithms.

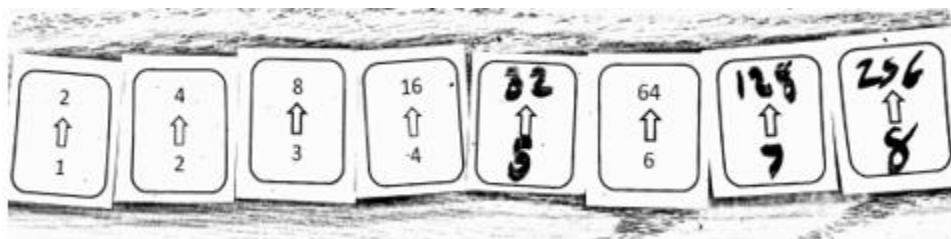


Figure 7. Lesson 1: Base 2 Card Activity

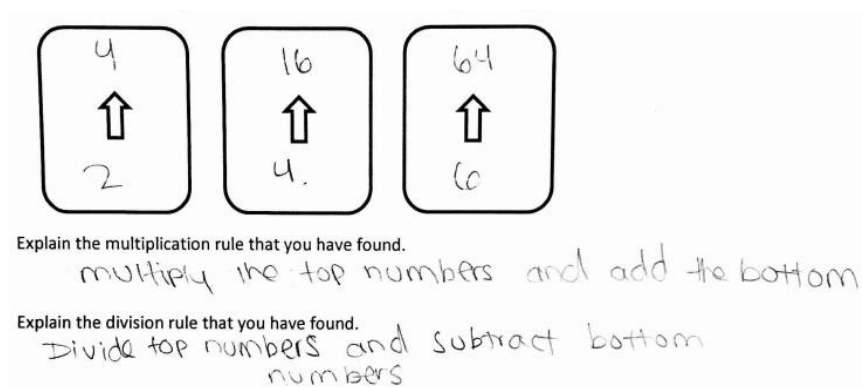


Figure 8. Lesson 1: Base 2 Card Activity (Multiplication rule)

As the Base 2 Card Activity continued, students were asked to create more cards for the game. They extended the cards (Figure 7) in both directions and further developed their understanding of the properties of exponents and logarithms through a numerical representation. The students articulated their understanding of the sequences on the top and bottom of the cards. They understood that when moving to the right the bottom of the card is added by one and the top of the card is multiplied by two and moving to the left would subtract one from the bottom and divide two on the top. One group was observed developing their understanding of negative exponents and reciprocals. They stated that two is paired with four and negative two is paired with one fourth (Figure 9).

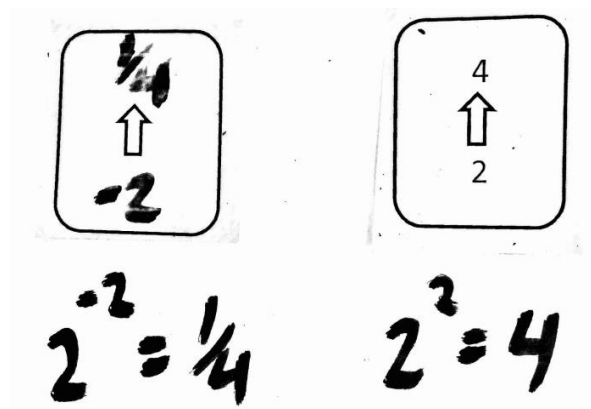


Figure 9. Example of negative exponent rule

Note: Not students work, but an example of the observed notes.

Other groups were observed developing the idea that $2^n > 0$ and for $\log_n x$, $x > 0$ through a numerical representation. During the activity, students were asked if several new cards would follow the rules created for the game. The card “ $__ \rightarrow -4$ ” was quickly eliminated by the groups. For example, when asked about the “ $__ \rightarrow -4$ ” card, Cailin stated that “you can’t divide 2 by anything to get -4” (Figure 10). In addition, Jay and Jim explained the idea that $2^n > 0$ and $x > 0$ for $\log_n x$ by discussing the card “ $-\infty \rightarrow __$ ”. First, the two students understood that moving to the left on the sequence of cards meant that the bottom is subtracted by one and the top is divided by 2. Then they explained that you would have to divide 2 forever to reach negative infinity. When asked for the opposite side of the card “ $-\infty \rightarrow __$ ”, they put zero (Figure 11).

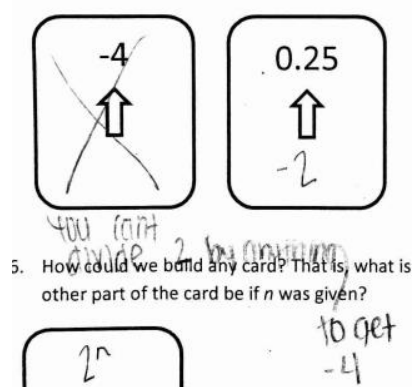


Figure 10. Student work: Cailin

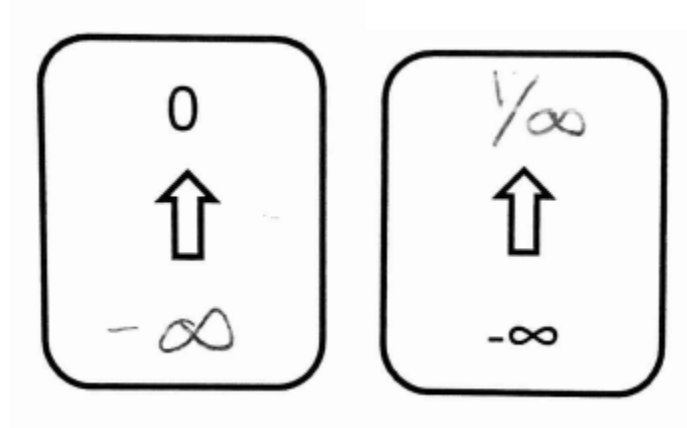


Figure 11. Student work: Jayden and Jim

After extending the number line, students were asked to algebraically represent the functional relationship on the cards. Like the control group, students struggled with this correspondence view of the relationship. Once shown that the function was $f(x) = 2^x$, the treatment group made a connection back to the sequence. They understood that the base was two because you are multiplying by two to move to the right on the top of the cards.

Once students understood the functional relationship, the treatment group was also able to extend their knowledge of powers of two to non-integer exponents. When looking at a given sequence of cards (Figure 12), students concluded that the card “ $\frac{1}{2} \rightarrow$ ” was associated with the square root of two. For example, Wanda stated that the bottom of the cards “increase by double” while in the top row “the number to the left is the square root of the number on the right.” Therefore, the highest observed level of understanding in the “Base 2 Card” activity was “Generalization” (Weber, 2002). Students were able to explain the value $2^{\frac{1}{2}}$ as the square root of 2.

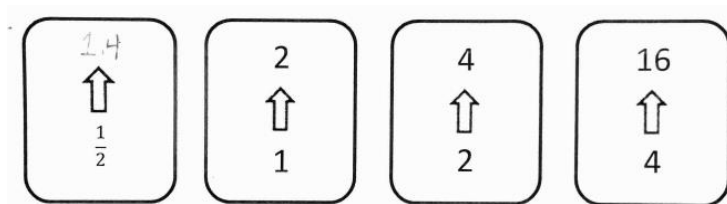


Figure 12. Students work: Wanda

Students in the treatment group were also allowed to view this relationship graphically (Figure 13). After dealing with the cards and extending the cards in both directions, students were asked to graph the relationship. They made a table based on the cards that they created and graphed the function that expressed the top as a function of the bottom ($f(x) = 2^x$) and the function that expressed the bottom as a function of the top ($f(x) = \log_2 x$). (The students were

not shown the logarithmic notation, but only asked to graph the inverse by using the top of the cards as the domain and the bottom as the range.) Through this graphical representation, students were able to transfer their knowledge of the numerical representation and make a connection to the asymptotes of the graphs. For example, Jim noticed that the graphs approached one of the axes and stated that the top of the cards are approaching zero when moving to the left. In addition, Torrence described the asymptote as a limit. Overall, students in the treatment group concluded that the top of the cards were approaching zero when moving to the left and made the connection to the graph when led in a discussion by the instructor.

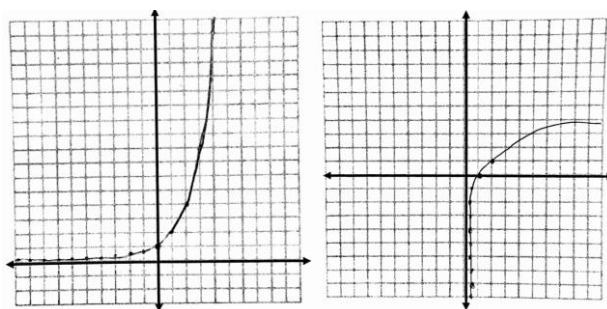


Figure 13. Student Work: Jane

In summary, lesson 1 provided the treatment group with the opportunity to reach a deeper level of conceptual understanding of logarithms (Table 8). Students in the treatment group were provided the opportunity to think at the “Generalized” level of understanding through covariational reasoning while the control group was only able to reach the “Exponentiation as a process” level through the correspondence approach. Through the first lesson, the difference in the levels of conceptual understanding present in each student is not clear, but the opportunity for a greater understanding of exponential and logarithmic functions was available to the treatment group through the Base 2 Card activity. For example, the treatment group had the opportunity to develop their knowledge of the properties of exponents and logarithms through using a

covariational approach. In addition, both groups represented the powers of two numerically, algebraically, and graphically. However, the treatment group was more frequently observed making deeper connections among different representations. For example, both the control and treatment groups noticed that the function $f(x) = 2^x$ was increasing based on the table and graph, but students in the treatment group were connecting the asymptote in the graph to the table by extending the domain of the table and graph toward negative infinity while the control group was limited by their task to the natural number exponents and first quadrant on the coordinate plane. In addition, students in both the control and treatment groups were inherently comfortable and easily recognized the patterns in the table using covariational reasoning (Confrey & Smith, 1994) and both groups struggled using the correspondence approach to develop a function from the tables.

Table 8:

Lesson 1 counts of conceptual understanding of logarithms

<u>Levels of Conceptual Understanding (Weber, 2002)</u>	<u>Control (n=17)</u>	<u>Treatment (n=23)</u>
Exponentiation as an Action	17 (100%)	23 (100%)
Exponentiation as a Process	17 (100%)	23 (100%)
Exponential Expressions as the Result of a Process	0 (0%)	20 (87%)
Generalization	0 (0%)	15 (65%)

Lesson 2. The control group participated in the “Graphs of Exponential Functions” task (GSE, 2015), which required three class periods of 50 minutes. Students were expected to use a correspondence approach (Confrey & Smith, 1995) to investigate the graphs of exponential functions with different bases. They were expected to understand, using technology, the

transformations of the graphs of exponential functions. The treatment group participated in the “Different Bases” task (Appendix E), which required two class periods of 50 minutes. Students were expected to use covariational reasoning (Ferrari-Escolá et al., 2016) to develop their knowledge of exponential functions with different bases. Again, the students in the treatment group were expected to discover various properties of exponents and logarithms. Each task built on the knowledge gained from the previous tasks that investigated the function $f(x) = 2^x$.

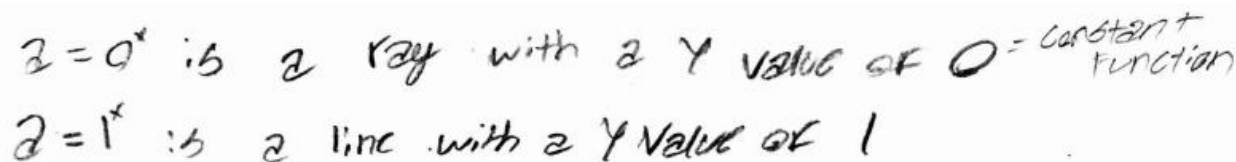
In the control group, students were first asked to graph the function 2^x , 3^x , 4^x , and 10^x , and assess the similarities and differences between the graphs by describing different characteristics of the graphs such as domain, range, interval of increase, interval of decrease, asymptote, and end-behavior. Through the graphical representation, students noticed that the graphs of the functions were increasing, and the y-intercepts were all one. For example, Eddie stated, “they all go through the same place on the y-axis,” and Sally stated, “they are all increasing.” When examining the differences of the graphs, students referred to the rate of change of the graphs. For example, Andrew observed that the graphs became steeper. Conversations between the teacher and the students led to some noticing that the base effected the steepness of the graph. Karl stated, “the higher the number, the steeper.” During this part of the task, students were able to gain a “process understanding of exponents” (Weber, 2002). In addition, students needed a review of the terms such as domain, range, ect. to use those terms when describing the graphs.

As the control group progressed through the “Graphs of Exponential Functions” task (GSE, 2015), students were asked to use a graphing calculator to graph the functions 2^{-x} , 3^{-x} , 4^{-x} , and 10^{-x} and compare the graphs to the functions 2^x , 3^x , 4^x , and 10^x . Students compared the graphs by referring to the transformation as a reflection. For example, Sally stated “the line [sic]

is going in the opposite direction. They are reflections.” In addition, students were to compare the function $\left(\frac{1}{2}\right)^x$ and 2^{-x} . The students were able to use a graphical representation to show that the functions were the same, but they could not on their own use algebraic representations to show that they are the same. After some coaching and showing the students the negative exponent property, some students gained an understanding that $\left(\frac{1}{2}\right)^x$ and 2^{-x} were the same function. For example, Arthur stated “1/2 is basically like saying your dividing by 2, so if the 2 has a negative exponent it’s almost like dividing.” During this part of the task, students in the control group were able to reach the “exponential expressions as the result of a process” level of understanding (Weber, 2002) by using exponential properties to prove equality.

At the end of the second lesson, the control group was given that an exponential function is of the form $y = a^x$ for $a > 0$ and $a \neq 1$. Then they were asked why. To explain why a could not be negative, the students were observed trying to graph on the graphing calculator an exponential function with a negative base. Because the equation would not graph, students concluded that the base could not be negative. For example, Hollie stated “your graph will not be a line[sic] if your base is negative.” The justification of their answer was based on the error message provided by the graphing calculator and not an understanding of the base. In addition, students were asked why the base of an exponential function could not be zero or one. Again, students were observed graphing 0^x and 1^x . The only answer that students provided was that the graphs were not exponential but were constant (Figure 14). Students in the control group did not make a connection between the graph and algebraic representation of the base zero and base one functions beyond an association between the graph and the function. They did not express an understanding of the meaning of the base in the exponential functions to explain the behavior in the graph. Therefore, the level of understanding reached by the students in this part of the task is

unclear. The justification for their answers were based on the graph of exponential equations and not on an understanding that exponents represent repetitive multiplication of the base. After the lesson, the instructor led a class discussion about the base of an exponential function and explained why the base is greater than zero and not one. If students were able to internalize this understanding, then they reached a “process understanding of exponents” (Weber, 2002)



$2 = 0^x$ is a ray with a y value of 0 = constant function
 $2 = 1^x$ is a line with a y value of 1

Figure 14. Student work: Mason and Nemo

In the treatment group, students were given base three, four, and ten cards (Figure 15). Each set of cards represented the functions 3^x , 4^x , and 10^x respectively. From these cards, students were asked several questions about what happens to the top of the cards if the bottom of the cards are added, subtracted, multiplied (repetitively added), zero, multiplied by half, and multiplied by negative one. The purpose of the task was to show the relationship between the arithmetic sequence with a common difference of one and a geometric sequence with different common ratios have the same properties. Because students already discovered the multiplication and division rule from the Base 2 Card activity, they quickly recognized that the same rules apply to the different sets of cards. In addition, students were able to observe that the card “ $0 \rightarrow 1$ ” was in each set and concluded that zero in the arithmetic sequence is associated with one in the geometric sequence. Through further manipulation of the cards, students discovered various properties of exponents and logarithms without use of the notation (Figure 16). Students were able to reach the “Generalization” level of understanding, because students could explain the meaning of the half power as the square root (Weber, 2002).

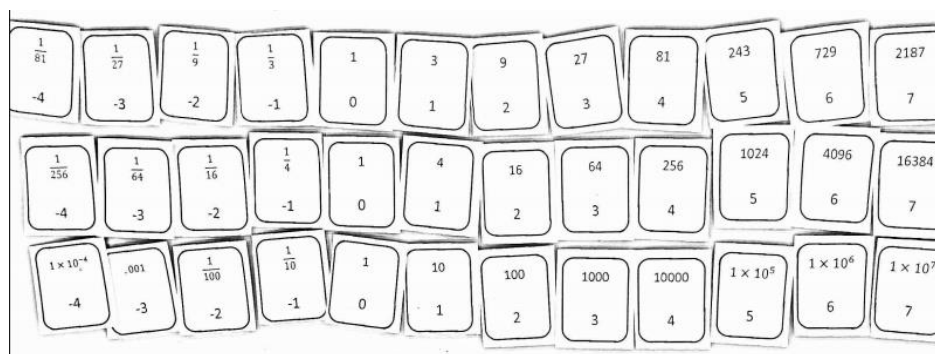


Figure 15. Different Bases Task cards

When I am _____ in the Adding world (bottom row of numbers)...., I am _____ in the Multiplying world (top row of numbers)	Draw and Example
Adding	Multiplying	$\begin{array}{ c } \hline 100 \\ \hline 2 \\ \hline \end{array} \times \begin{array}{ c } \hline 1000 \\ \hline 3 \\ \hline \end{array} = \begin{array}{ c } \hline 1 \times 10^5 \\ \hline 5 \\ \hline \end{array}$
Subtracting	dividing	$\begin{array}{ c } \hline 1 \times 10^5 \\ \hline 5 \\ \hline \end{array} \div \begin{array}{ c } \hline 1000 \\ \hline 3 \\ \hline \end{array} = \begin{array}{ c } \hline 100 \\ \hline 2 \\ \hline \end{array}$
Multiplying (repetitive addition)	exponents (powers)	$\begin{array}{ c } \hline 4 \\ \uparrow \\ 1 \\ \hline \end{array} \begin{array}{ c } \hline 4 \\ \uparrow \\ 1 \\ \hline \end{array} \begin{array}{ c } \hline 4 \\ \uparrow \\ 1 \\ \hline \end{array} = \begin{array}{ c } \hline 64 \\ \uparrow \\ 3 \\ \hline \end{array}$
Zero	one	$\begin{array}{ c } \hline 1 \\ \hline 0 \\ \hline \end{array}$
Multiplying by $\frac{1}{2}$	Square root	$\begin{array}{ c } \hline 81 \\ \uparrow \\ 4 \\ \hline \end{array} \times \frac{1}{2} = \begin{array}{ c } \hline 9 \\ \uparrow \\ 2 \\ \hline \end{array}$
Multiplying by -1	reciprocal	$\begin{array}{ c } \hline 81 \\ \uparrow \\ 4 \\ \hline \end{array} \times (-1) = \begin{array}{ c } \hline \frac{1}{81} \\ \uparrow \\ -4 \\ \hline \end{array}$

Figure 16. Students work: Casey

The next section of the “Different Bases” task required the treatment group to graph each set of cards by treating the bottom as the domain and the top as the range, and then graphing each set by treating the top as the domain and the bottom as the range. (Students knew that the relationship between mapping the bottom to the top is 3^x , 4^x , and 10^x , but they had not seen the logarithmic notation to know the function of the inverse mapping.) Like the control group, students in the treatment group needed a review of the characteristics of graphs. After the whole class discussion, many students were able to find the domain, range, and asymptote of each of

the six functions, and they were able to articulate that the exponential function and their inverse had similar characteristics that were “opposite”. For example, Carson stated that the asymptote is zero for the function $f(x) = 3^x$, because “there is always something left over to divide into 3 parts.” In addition, John determined that the graph of the inverse of an exponential function could not be in the third quadrant because “dividing by a number will never reach zero and therefore will never be negative.” Therefore, some students in the treatment group were able to reach the “Exponentiation as a process” level of understanding, because they were able to reason about the properties of the function (Weber, 2002). Though students in the treatment group needed a review of the characteristic of a graph, they were able to articulate their understanding of those characteristics.

At the end of the “Different Bases” task, students in the treatment group were introduced to the logarithmic notation. They were shown that each set of cards could be expressed algebraically as either an exponential function ($m = b^a$) or a logarithmic function ($a = \log_b m$). (The variables “a” and “m” were used because the exponent was described as the adding world and the result was described as the multiplying world.) After the instructor showed the exponential and logarithmic functions with a base of two, students were given the opportunity to write the exponential and logarithmic functions with a base of three, four, and ten. As students were working, the instructor went to each group and asked the question “what is the meaning of the base?” Without any coaching, several groups were able to articulate the meaning of the base. For example, Jesse stated, “because that is what we are multiplying by” and Jake stated, “it multiplies by three, four, and ten.”

Through lesson two, students in both the control and treatment group were provided opportunities to use graphical representations to describe exponential functions and their

characteristics. The control group was given the opportunity to reach the “Exponential expressions as the result of a process” level of understanding (Weber, 2002) when comparing the graphs of $\left(\frac{1}{2}\right)^x$ and 2^{-x} , and the treatment group was able to reach a “Generalization” level when developing an understanding of the half power. Again, the treatment group had the opportunity to reach a deeper level of understanding exponential and logarithmic functions during lesson 2 (Table 9). In addition, both groups required an overview of characteristics of graphs and were able to use those characteristics to describe graphs. However, the treatment group showed that they could transfer their knowledge of the numerical representation of the exponential functions to the graph when describing asymptotes and algebraic representations of the functions to the graphs when articulating the meaning of the base. The control group made connections between the algebraic and graphical representations using technology, but they could not articulate the relationship beyond an association between the function and the graph. By this point in the unit, the control and treatment groups have represented exponential functions numerically by using a table, algebraically by being shown or writing the function, and graphically. However, the treatment group has investigated properties of exponential functions, while the control group has investigated transformations of exponential functions.

Table 9

Lesson 2 counts of conceptual understanding of logarithms

<u>Levels of Conceptual Understanding (Weber, 2002)</u>	<u>Control (n=17)</u>	<u>Treatment (n=23)</u>
Exponentiation as an Action	N/A	23 (100%)
Exponentiation as a Process	17 (100%)	23 (100%)
Exponential Expressions as the Result of a Process	6 (35%)	20 (87%)*
Generalization	0 (0%)	20 (87%)*

Note: The “Graphs of Exponential Functions” task did not require the Control group to express an “Exponentiation as an Action” level of understanding.

**The instructor led the treatment group through a class discussion on the properties of exponents and logarithms. Therefore, it is unclear if the students discovered the properties on their own (Generalization) or with coaching (Exponential Expression as the Result of a Process.)*

Lesson 3. By this time in the integrated STEM and in the traditional units, both the treatment and control group had participated in the Cabbage Juice Lab during chemistry in which red cabbage juice, a natural pH indicator, was used to measure the pH of various household chemicals. Students observed that the color of the cabbage juice changed when mixed with other chemicals. Therefore, during lesson three, the treatment group was given the opportunity to expand on the cabbage juice lab while the control group applied their knowledge of exponential functions to bacteria growth. The control group participated in the “Bacteria in the Swimming Pool” task (GSE, 2015), which required one class period of 50 minutes. The researcher expected the control group to use a correspondence approach (Confrey & Smith, 1995) to investigate and model a situation in which 1500 bacteria per cc was found in a swimming pool and is expected to double each day. The task led students to understanding the need for the inverse of an exponential function. The treatment group participated in the “pH and Logarithms: Base 10 with

a twist” activity (Appendix F), which required two class periods of 50 minutes. The researcher expected the treatment group to use covariational reasoning (Ferrari-Escolá et al., 2016) to develop their knowledge of logarithms in a science context.

In the control group, students read the scenario about the bacteria growth and then they calculated the number of bacteria expected after two days, filled in a table for the first 5 days, and estimated the number of bacteria expected after 3.25 days. To calculate the bacteria after two days, the researcher observed groups multiplying the initial amount (1500 bacteria) by two twice ($1500 \times 2 \times 2$). In addition, the table was filled in by multiplying the previous bacteria count by two. To estimate the expected bacteria count after 3.25 days, all groups gave a value between 12000 (3 days) and 24000 (4 days). When asked to explain their answer, many groups knew that it would be between the three and four day amount, and they gave similar explanations to how they came to their conclusion. For example, Karl stated “I went up to 3 and .25 of it to its self” to get 15000 ($12000 + 0.25 \times 12000$). Therefore, students showed that they could function at an “Exponentiation as an action” understanding of exponents (Weber, 2002), because they could calculate bacteria counts based on whole number exponents.

As the “Bacteria in the Swimming Pool” activity progressed, the teacher asked students to develop an equation that models the situation, and then calculate the number of bacteria after 3.25 days. Like in lesson one, students had a difficult time developing the equation with a covariational approach. For example, the instructor observed that Arthur wrote 2^x on his paper. When asked why, he stated that he thought it would be an equation like it. When the instructor returned later, Arthur had written 3^x . Therefore, Arthur had not developed an understanding of the base of an exponential equation. After further inquiry by the instructor, he was plugging in values into his equation to see if the output was the expected value in the table. No group was

able to develop the mathematical model representing the bacteria growth as a function of time ($A(t) = 1500(2)^t$) until after coaching by the instructor. The instructor, using the table, showed that the initial amount of bacteria is 1500 and it is doubling each day (day 1: $1500 \cdot 2$; day 2: $1500 \cdot 2 \cdot 2$, day 3: $1500 \cdot 2 \cdot 2 \cdot 2$, and so on). Because of the discussion, Cheer wrote the expression $1500(2)^x$, and then plugged in 3.25 to calculate the number of bacteria after 3.25 days.

After developing the equation to model the situation, the teacher instructed students to graph the model and use the model to predict the number of bacteria after 3.25 days and predict the number of days for the bacteria to grow to three million. When asked how they would use the graph to predict the amount of bacteria after 3.25 days, students answered by stating that the trace feature of the graphing calculator be used to find when x is 3.25. Students did transfer their knowledge of the algebraic representation to the graphical representation. They understood that the input and output of the function are the coordinates of the graph. To find the number of days the bacteria needed to grow to three million, many groups used the guess-and-check method. They first guessed a value and then increased or decreased the number of days to get close to three million. The conclusion by most groups was about 11 days. From this problem, the instructor showed the class the logarithmic notation and how it can be used to solve exponential equations. Therefore, the control group was able to reach the “Exponentiation as a process” understanding of exponents, because they were shown how to reverse the process of exponents.

In the treatment group, students first arranged pH cards (Figure 17) in order, then they made observations about the relationship between pH and the concentration using a covariational approach. The instructor did not direct the students to arrange them in order based on the pH or the Hydrogen ion concentration, but most arranged them from least to greatest using the pH

values. When asked what happens to the concentration as the pH increases, most participants stated that the concentration decreases. Some added to that statement by giving the rate at which it changes. For example, Patrick stated “it is getting smaller ... by factors of 10.” Next, participants made observations about what happens to the concentration when the pH is increased by one. Most groups noticed that the concentration was divided by ten.

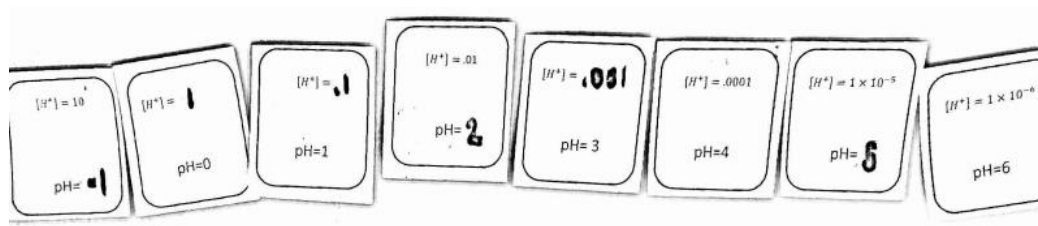
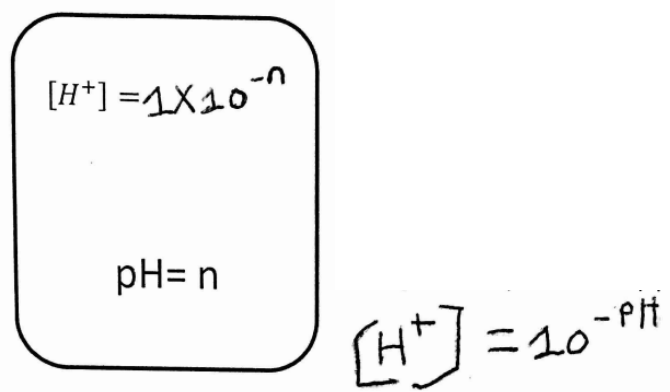


Figure 17. pH cards

To further explore the relationship between pH and hydrogen ion concentration, students were asked to determine if a solution with pH of 7.6 or a pH of 3.6 had greater concentration and by how much? From the previous two questions, most groups concluded that the pH of 3.6 had a greater concentration, but only a few groups were able to articulate the relative difference between the concentrations without guidance. However, through discussion with the instructor, most groups concluded that the solution with a pH of 3.6 is 10000 times more concentrated. Students based this decision on the rate at which the concentration changes (divide by 10) as the pH increases by one. This section of the task allowed students to build on their knowledge of exponential and logarithmic relationships and develop their conceptual understanding of pH through numerical representations. In addition, students were able to reach the “Exponentiation as a process” level of understanding (Weber, 2002), because they were using negative exponents and discovering properties of an exponential and logarithmic relationship.

After investigating pH with a numerical representation, students in the treatment group were asked to develop an exponential and logarithmic model for pH and hydrogen ion

concentration and investigate properties of the relationship. Because students had created exponential models for a base 2, 3, 4, and 10 systems by using the cards from previous activities, they understood that the function that maps pH to hydrogen ion concentration $[H^+]$ was $[H^+] = 10^{-pH}$ (Figure 18). However, they did struggle with creating a logarithmic function that maps $[H^+]$ to pH. Some students understood that the base was 10. When presented with the function $pH = -\log[H^+]$ and asked why the base was ten, Billy stated “you are multiplying and dividing by ten.” In addition, when asked why the function contained a negative, Jamie stated “as pH increases, concentration decreases.”



The image shows handwritten student work. On the left, there is a rounded rectangular box containing the equation $[H^+] = 1 \times 10^{-n}$ at the top and $pH = n$ at the bottom. To the right of the box, the equation $[H^+] = 10^{-pH}$ is written.

Figure 18. Student work: Noah

To extend their knowledge of pH, students were also asked to develop properties for the pH relationship using a covariational approach. Like in the previous task, students were directed to perform an operation with the bottom of the card (pH) and observe the effect on the top of the card ($[H^+]$). Some groups were able to discover that adding one or two to the pH would change $[H^+]$ by dividing it by 10 or 100 respectively, but other groups came to this realization with some coaching from the instructor (Figure 19). Therefore, with the numerical representation becoming familiar to students, they could transfer their knowledge of these sequences from a numerical representation to an algebraic representation more easily. In addition, the transfer of their

knowledge from a numerical to an algebraic relationship allows the students to reach the “Exponentiation as a process” level of understanding (Weber, 2002).

Fill out the chart below using the cards created

When I _____ to the pH... (Adding world)	..., $[H^+]$ is _____ (Multiplying world)	Draw an Example
Add one	Divide by 10 or Multiplied by $1/10$	10^{-2} \uparrow pH=2 $\div +1 = 10^{-3}$ pH=3
Subtract one	Multiply by 10	$[H^+] = .01$ \uparrow pH=2 $\times 10 = .1$ pH=1
Add two	divide by 100	$[H^+] = .1$ \uparrow pH=1 $\div 100 = .001$ pH=3
Subtract 2	Multiply by 100	$[H^+] = .001$ \uparrow pH=3 $\times 100 = .1$ pH=1
When the pH is zero...	... the $[H^+]$ is <u>1</u> .	$[H^+] = 1$ pH=0

Figure 19. Student work: Alice

After representing the pH and $[H^+]$ relationship numerically and algebraically, students were asked to graph the logarithmic relationship by using $[H^+]$ as the domain and pH as the range and discover certain characteristics of pH and $[H^+]$. Students did struggle with graphing this relationship, because it seemed to be the first time they have been asked to graph coordinates with scientific notation such as $(10^{-5}, 5)$. For example, several groups asked the instructor how they could graph numbers that are extremely small. However, some groups understood certain characteristics of the graph such as the asymptote. For example, Carson and Cailin used the table to show that as pH increases the $[H^+]$ approaches zero. They stated “[H^+] goes to zero as pH increases, because pH increases by one, $[H^+]$ is divided by 10. And by dividing by 10, you will never reach zero.” In addition, students used both the table and the graph to show that when the

pH is zero the concentration is one, that pH can be negative if the $[H^+]$ is greater than one, that pH has a range of negative infinity to infinity (according to the mathematical model), and that $[H^+]$ can never be zero (Figure 20).

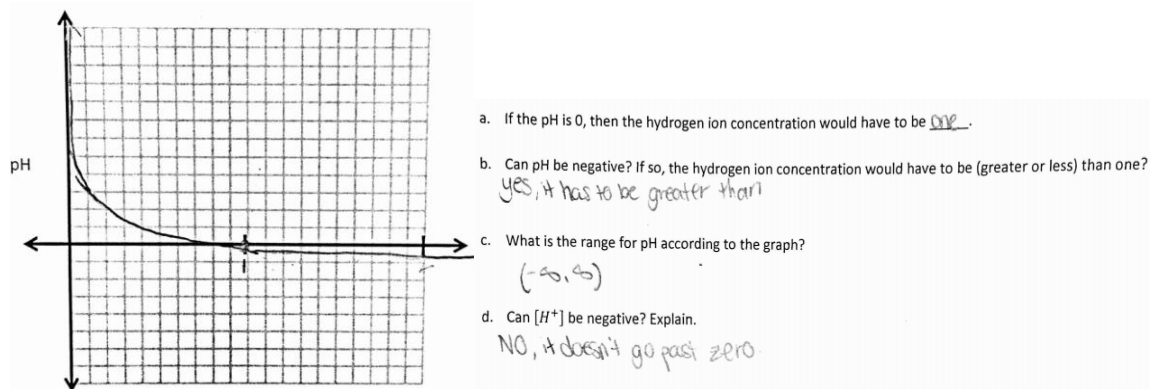


Figure 20. Student work: Sky

During the “pH and Logarithms: Base 10 with a twist” task, students were also transferring their knowledge of pH from chemistry. For example, students were asked what the typical range of pH in chemistry is. 15 out of 23 (65%) of the students stated either 1 to 14 or 0 to 14. However, in the task, students found that the range of pH is from negative infinity to infinity according to the mathematical model. Jeanine stated that she was confused by the fact that in chemistry pH goes from 0 to 14, but pH in math goes from negative infinity to infinity. This inconsistency gave rise to progressive conversations about the meaning of concentration such as why the real-world context of pH does not have a boundless range like the logarithmic equation suggest and why pH is not limited to the typical range (0 to 14) taught in Chemistry.

During lesson three, both the control and treatment group were exposed to applications of exponential and logarithmic functions. They each had the opportunity to create mathematical models to represent real-life scenarios and think at the “Exponentiation as a Process” level of understanding exponential and logarithmic functions (Table 10). Both groups furthered their

understanding of the components of the mathematical models. However, the treatment group seemed to have a greater understanding of the components of the exponential equation. For example, they were able to articulate the meaning of the base and use that understanding to create mathematical models. In addition, the treatment group was better able to transfer their knowledge from one representation to another, because they had been exposed to a consistent way of dealing with these relationships through covariational reasoning. Students in the control group had to be exposed to a covariational approach before they were able to develop their mathematical model for the bacterial growth. In addition, the treatment group benefited from an STEM integrated approach, because they were able to investigate the relationship between pH and $[H^+]$ to further their conceptual understanding of pH. They could move beyond an “object” domain of understanding of pH to a “Functional” domain of understanding (Park & Choi, 2012), because they connected the qualitative aspect of the Cabbage Juice Lab from chemistry to the mathematical representation of pH in the mathematics classroom.

Table 10

Lesson 3 counts of conceptual understanding of logarithms

<u>Levels of Conceptual Understanding (Weber, 2002)</u>	<u>Control (n=17)</u>	<u>Treatment (n=23)</u>
Exponentiation as an Action	17(100%)	23 (100%)
Exponentiation as a Process	17 (100%)	23 (100%)
Exponential Expressions as the Result of a Process	N/A	N/A
Generalization	N/A	N/A

Note: Both task did not require students to think beyond an Exponentiation as a process level of understanding.

Lesson 4. The researcher used lesson four to further develop the participants understanding of the logarithmic notation in both the control and treatment groups. The control

group participated in the “What is a Logarithm?” task (GSE, 2015), which required two class periods of 50 minutes. In this task, participants were further exposed to the logarithmic notation and asked to use that notation to convert exponential equations to logarithmic equations, evaluate logarithmic expressions, and solve exponential and logarithmic equations. The treatment group was given a practice worksheet, which required one class period of 50 minutes, that allowed them to convert an exponential equation to a logarithmic equation and evaluate a logarithmic expression using a covariational approach.

In the control group, students were shown that inverse operations exist for both mathematical operations and non-mathematical operations such as opening and closing a door. Students were led through this discussion to help them understand the inverse relationship between exponents and logarithms. After the discussion, students were exposed to the exponential equation $a = b^c$ and the logarithmic equation $\log_b a = c$, and they were shown a few examples of equations in both exponential and logarithmic form. Then they were given the opportunity to convert an equation to different forms and evaluate logarithmic expressions. The students were able to convert equations from one form to another. In addition, the students were able to evaluate logarithms in which the solution was a positive integer. However, students struggled and needed further coaching over exponent properties to evaluate logarithmic expressions in which the solution was a negative integer or a rational number. For example, Kari asked for a calculator and stated, “going from a smaller number to a bigger number was easier.” He understood how to evaluate logarithmic expressions such as $\log_4 64$, but he could not think of how to evaluate logarithmic expressions such as $\log_{27} 9$ or $\log_4 \frac{1}{16}$ without a calculator. The instructor, using a correspondence approach, did review the properties of exponents such as the negative exponent property to evaluate a logarithmic expression such as $\log_4 \frac{1}{16}$. However, the

instructor could only explain how to evaluate logarithmic expressions such as $\log_{27} 9$ by using rational exponents and radicals. After the review, students were able to evaluate a logarithmic expression that had integer solutions, but most still struggled with logarithmic expressions that had rational solutions.

After evaluating logarithmic expressions and converting forms, students in the control group were directed to solve logarithmic and exponential equations. First, students were able to transfer their knowledge of evaluating logarithmic expressions to solve logarithmic equations. For example, Cory cubed eight to solve the equation $\log_8 x = 3$, because he knew that $8^3 = x$. In addition, Trevor solved the equation $\log_6(4x - 7) = 0$ by showing that 6^0 is one. Therefore, he concluded that $4x - 7$ must equal one. Second, students were asked to solve exponential equations such as $10^x = 15$ using a calculator, because the solution could not be easily evaluated without technology. The control group solved the exponential equation by converting the equation to a logarithmic form and evaluating the logarithmic expression in the calculator (Figure 21). However, when solving the equation $2(10^x) = -6538$, students (12 out of 17) were confused by the error message in the calculator. They thought that they did something wrong.

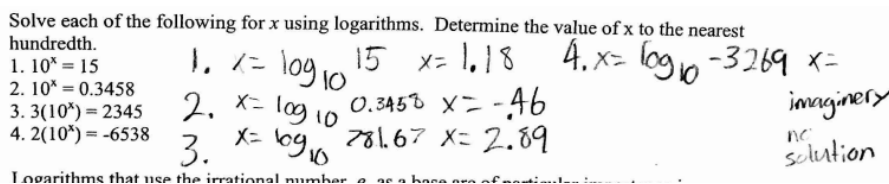


Figure 21. Student work: Cheer

For example, Cheer did not understand why the answer was no solution. In a conversation with the instructor, his misconception was revealed.

Cheer: Why does the calculator give me an error?

Instructor: "Does the error message make sense? Why might you get an error message?"

Cheer: “I do not know.”

Instructor: “What is the base of the power?”

Cheer: “10”

Instructor: “Can 10 be raised to a power and the outcome be negative.

Cheer: “if the exponent is negative”

After explaining powers of 10, Cheer understood why the error message occurred. Therefore, the control group was able to solve exponential and logarithmic equations, but they had difficulty transferring their knowledge of the graphical representation and numerical representation to the algebraic representation to make sense of the equations.

The treatment group was again shown the general exponential equation ($m = b^a$) and the general logarithmic equation ($a = \log_b m$), where a represents the bottom of the cards (adding world) and m represents the top of the cards (multiplying world). (The phrases multiplying world and adding world refer to the card activities.) After a brief discussion of the notations, the treatment group converted exponential equations to logarithmic and logarithmic equations to exponential. Like the control group, the treatment group could convert these equations from one form to the other. Therefore, both the treatment and the control group could algebraically represent exponential and logarithmic equations in different forms.

After converting forms, the treatment group was given the opportunity to evaluate logarithmic expressions. Like the control group, the participants were able to evaluate logarithmic expressions that were whole numbers such as $\log_4 64$. However, participants in the treatment group were also able to transfer their knowledge of the numerical representation of the cards to the algebraic representations of the logarithmic expressions and evaluate logarithms that were negative integers such as $\log_4 \frac{1}{16}$. The treatment group also struggled with evaluating

logarithmic expressions with rational answers, but they were able to more easily learn how this type of logarithmic expression is calculated by referring to the numerical representation of the cards and using covariational reasoning. The instructor introduced the double number line and examples of how to use the double number line were shown (Figure 22). The instructor explained that the top of the double number line is a geometric sequence that progresses by multiplying by a number (the base) and the bottom of the double number line is an arithmetic sequence that is being added. For example, to evaluate $\log_{64} \frac{1}{4}$, one must draw a base 64 system, in which the top of the number line is being multiplied by 64 when moving to the right and dividing by 64 when moving to the left. The system starts at zero in the adding world and one in the multiplying world. The instructor goes on to explain that if the argument cannot be found in this way, then the system must be broken into smaller multiplying steps (foundation for the change of base formula). For example, a base 64 system can be broken into a base four system. Therefore, $\log_{64} \frac{1}{4} = -\frac{1}{3}$.

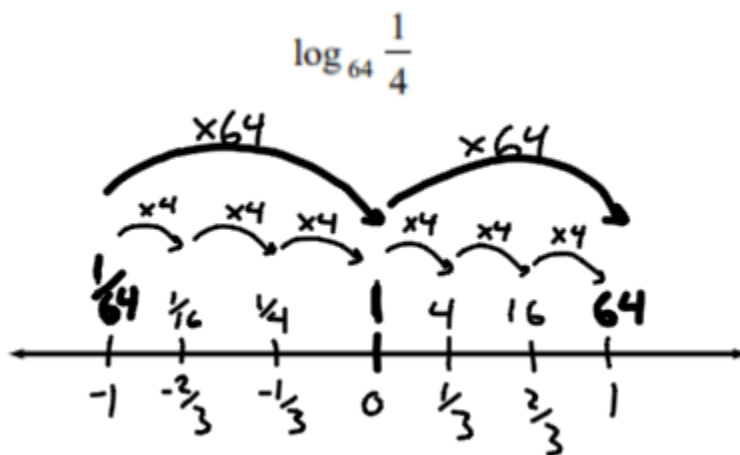


Figure 22. Instructor notes: Double number line example

From this example, most participants (17 out of 23) in the treatment group were able to evaluate logarithmic expressions that involve rational solutions by using a double number line

(Figure 23). In addition, some students were able to evaluate logarithmic expressions with rational solutions mentally. For example, Robert stated “log base 4 of 32 is 2.5 because 4 squared is 16 and 4 cubed is 64, and if I multiply 2, I can go from 16 to 32 to 64. So the answer is 2.5.”

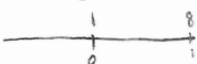
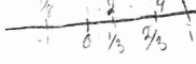
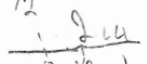
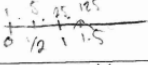
15] $\log_9 81 = 2$ $\log_9 9^2 = 2$	16] $\log_{27} 3 = \frac{1}{3}$	17] $\log_4 32 = 2\frac{1}{2}$
18] $\log_8 1 = 0$ 	19] $\ln e^4 = \log_e e^4 = 4$ * natural logarithm	20] $\log_8 4 = \frac{2}{3}$ 
21] $\log_3 \frac{1}{3} = -1$	22] $\log_{10} 1000 = 3$ $\log_{10} 10^3 = 3$ * common logarithm	23] $\log_{\frac{1}{2}} 128 = -7$
24] $\log_4 2 = \frac{1}{2}$ 	25] $\log_{25} 125 = 1.5$ 	26] $\log_3 \frac{1}{243} = -5$
27] $\log_4 64 = 3$	28] $\log_{64} 4 = \frac{1}{3}$	29] $\log_6 \frac{1}{216} = -3$

Figure 23. Student work: Lyn

Some students in the treatment group were able to extend their knowledge of exponents and logarithms by using the numerical representation of the double number line to discover properties that had not been previously discussed. For example, Lyn noticed that $\log_4 64$ and $\log_{64} 4$ were reciprocals. In addition, Lyn was able to show that $\ln e^4$ was 4 without formal instruction (Figure 23).

During lesson 4, both the control and treatment groups were provided the opportunity to convert between exponential and logarithmic representations of equations and evaluate logarithmic expressions. Both groups were able to convert expressions into both logarithmic and exponential forms. They showed that they understood the algorithm. However, the observational data showed that the treatment group had a better grasp of evaluating logarithms,

because they were able to think at a deeper level of understanding (Table 11). The control group was able to evaluate logarithms with whole number solutions but needed further instruction on evaluating logarithms with negative integer solutions or rational solutions. Even with instruction on exponent properties and the use of a correspondence approach, the observational data showed that most of the control group struggled evaluating logarithms with rational answers, but the students work showed the control group could evaluate logarithms with negative solutions. In contrast, the treatment group was able to evaluate logarithms with positive and negative integer solutions and were mostly able to evaluate logarithms with rational solutions after instruction using the double number line and a covariational approach. As a result, the control group could reach the “Exponential Expression as the Result of a Process” level of understanding, because they could evaluate logarithms that were not strictly whole numbers, but their understanding of negative powers and rational powers is connected to an arbitrary rule (Weber 2002). In contrast, the treatment group could reach beyond the “Exponential Expression as the Result of a Process” understanding and move toward a “Generalization” level of understanding, because their understanding of negative and rational exponents is not attributed to an arbitrary rule but a numerical representation (double number line).

Table 11

Lesson 4 counts of conceptual understanding of logarithms

<u>Levels of Conceptual Understanding (Weber, 2002)</u>	<u>Control (n=17)</u>	<u>Treatment (n=23)</u>
Exponentiation as an Action	N/A	N/A
Exponentiation as a Process	17 (100%)	23 (100%)
Exponential Expressions as the Result of a Process	17 (100%)	18 (78%)*
Generalization	0 (0%)	18 (78%)*

Note: Lesson 4 did not require the control or treatment groups to express an “Exponentiation as an Action” level of understanding.

**The instructor led the treatment group through a class discussion on double number line. Therefore, it is unclear if students were think at a Generalization level or Exponential Expression as the Result of a Process level of understanding.*

Lesson 5. During lesson five, both the control and treatment groups were asked to estimate powers of 10 such as $10^{2.3}$ and common logarithms such as $\log 500$ without a calculator. The purpose of the lesson was to allow students to become more familiar with a base 10 system that is commonly used in applications such as pH and to investigate if the control group would voluntarily transfer their knowledge from chemistry to mathematics. However, the treatment group was allowed to apply their knowledge to the pH value and $[H^+]$ for given $[H^+]$ and pH values respectively. The control group was instructed using a correspondence approach that focused on the properties of exponents, and the treatment group was instructed using a covariational approach that focused on the use of a double number line. The lesson required two 50-minute class periods.

The main difference in the lesson was the approach in which they were instructed. The participants in the control group used a correspondence approach to estimate logarithms between

two integers and they were instructed using previous knowledge of rational exponents to estimate logarithms to the nearest half. In other words, participants were reminded that the half power is equivalent to the square root operation. For example, participants were taught to estimate $\log 500$, by first understanding that $\log 500$ was between 2 (10^2) and 3 (10^3). Then they were shown $10^{0.5}$ power is 3.16 ($\sqrt{10} \approx 3.16$) and that 10^2 times $10^{0.5}$ is equal to $10^{2.5}$ using the product property of exponents. As a result, students could show that $10^{2.5}$ is 100 times 3.16 which is 316. Therefore, $\log 500$ is between 2.5 and 3, and a good estimation would be in-between those two values. The participants in the treatment group were able to estimate logarithms between two integers, and they were instructed that in a multiplying system halfway is equivalent to multiplying by the square root of the base. For example, in a times 10 system, to go from 1 to 2 to 3 you would multiply 10, but to go from 1 to 1.5 to 2 to 2.5 to 3, you would multiply by 3.16 ($\sqrt{10} \approx 3.16$) (Figure 24).

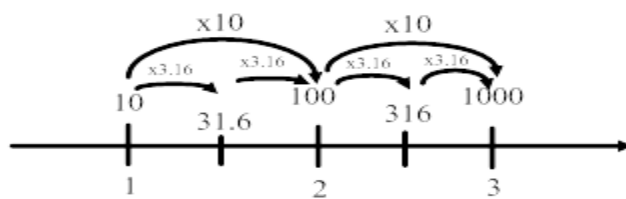


Figure 24. Double number line: Base 10

The students in the treatment group were able to reach a deeper level of understanding because their knowledge was not limited to memorized properties given by the teacher (Weber 2002). For example, Mason, a student in the control group, could only articulate his answer for powers of ten by using properties of exponents (Figure 25). The articulation of his answer was based on his knowledge of the product rule of exponents. However, Charlie, a student in the treatment group, articulated his answer through his understanding of a logarithmic scale (Figure

26). The articulation of his answer was based on his ability to interpret a situation in which a non-integer exponent was used.

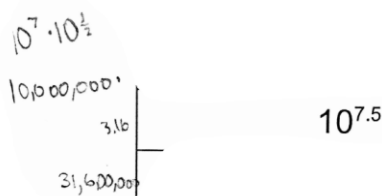


Figure 25. Student work: Mason

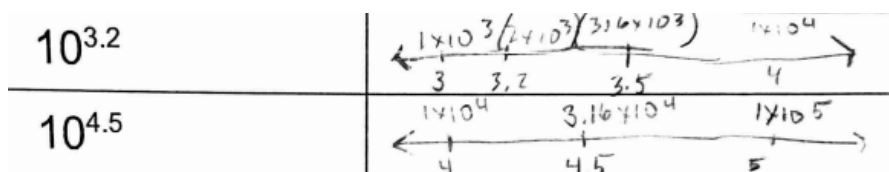


Figure 26. Student work: Charlie

In addition, the students in the treatment group were given the opportunity to deepen their level of understanding of pH. Instead of only estimating powers of ten and the common logarithms, students were asked to estimate pH given a $[H^+]$ and $[H^+]$ given pH. Again, students were able to articulate their understanding by using a logarithmic scale and not memorized rules. For example, Wanda was able to provide a detailed explanation for estimating the $[H^+]$ when the pH of a solution is 4.3 (Figure 27). First, she drew the logarithmic scale with pH's of 4, 4.5, and 5. In addition, she understood that as pH increases $[H^+]$ decreases. Using that knowledge, she understood the $[H^+]$ that needed to be associated with 4, 4.5, and 5. After drawing the double number line, she concluded that a good estimate for the $[H^+]$ of a solution with a pH of 4.3 was 5×10^{-5} .

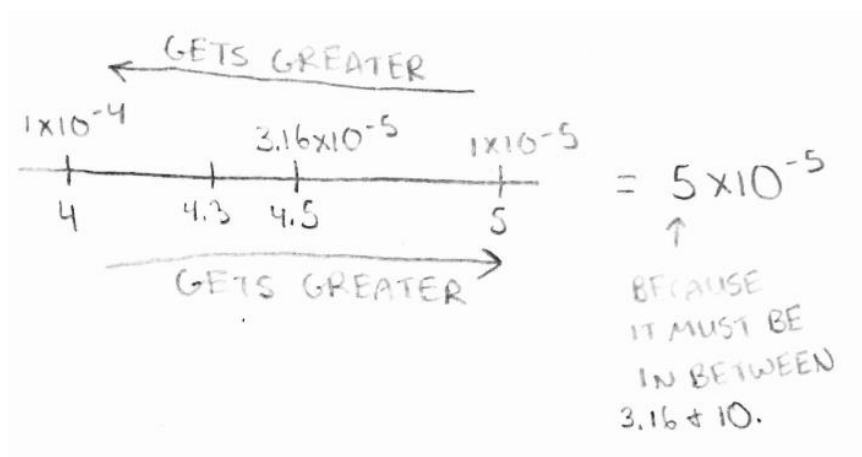


Figure 27. Students work: Wanda

However, students in the control group were not observed transferring their knowledge of chemistry to mathematics. In chemistry, students were asked to calculate pH values for given concentrations, which means they would be evaluating logarithms such as $-\log(3.5 \times 10^{-6})$. The difference between the problems experienced in chemistry and the problem in mathematics is the negative in front of the logarithm and the negative exponent. This observation does not mean that they could not transfer their knowledge, but it shows that students do not naturally volunteer the connections that they might see.

Lesson 6, 7, and 8: In lessons six, seven, and eight, students were given various problems that required them to understand properties of exponents and logarithms and solve exponential and logarithmic equations. In addition, they were given logarithmic and exponential expressions to estimate similar to lesson 5. Both groups were shown the algebraic representation of the exponent and logarithm properties, but the treatment group was reminded of and shown how they already discovered these properties using a double number line (Lesson 2).

Participants were asked to use properties of logarithms to fill in the blank of certain logarithmic expressions. When answering the question $\log_2(\quad) + \log_2(\quad) = \log_2(36)$, Josh, a student in the control group, stated “6 and 6”. When asked why, he stated “multiplication is

addition.” He was repeating a statement made by the instructor when explaining the logarithmic properties. In addition, when answering the question $-\log 8 = \log(\quad)$, Cory, a student in the control group, provided the answer one-eighth. When asked to explain, he had to glance at the board to find the property that he used. However, Caitlin, a student in the treatment group, explained, “I know that it is negative because it is a fraction and it will be left on the [double] number line.” Again, the student in the control was relying on memorized responses given by the instructor to justify his answers, and the student in the treatment group was relying on her understanding of the double number line. The student in the treatment group could articulate her answer without relying on a property written on the board or a statement shared by the instructor. Therefore, she was expressing a generalized understanding of logarithms and exponents (Weber, 2002) while the student in the control group was expressing at most “Exponential expressions as the result of a process” level of understanding.

Next, participants were asked to estimate logarithms with different bases such as $\log_8 89$. Similar results to lesson 5 were found. Students in the control group relied on properties of exponents while participants in the treatment group relied on the double number line. However, Mason, a student in the control group, did provide a different approach not seen in lesson 5 from the control group. When estimating $\log_8 89$, he stated that it was “between 2 and 3 because 8^2 is 64 and 8^3 is more than 89. It might be between 2 and 2.5, because 8^3 is so big and 89 is close to 64.” Mason narrowed his estimation for $\log_8 89$ by relying on intuition and not an understanding of a logarithmic scale. Though he was correct to estimate $\log_8 89$ to be between 2 and 2.5, he could not come to articulate the reasons for his response beyond his own intuition.

Next, students were asked to solve simple exponential equations such as $3^x = 17$. Students in the control solved similar equations in lesson 4 while students in the treatment group

had not been asked to solve these types of equations. In the control group, Marissa chose to estimate and provided that the answer was between 2 and 3, and Trevor converted the equation to $\log_3 17 = x$ and used that as his solution. Marissa was limited to integer exponents while Trevor used a property to convert the exponential equation to a logarithmic equation. Though Trevor's response was the exact answer, he could not explain why his answer was correct. In the treatment group, Jane approximated the solution to $3^x = 17$ to be between 2.5 and 3. When asked to explain, she argued that the answer had to be between 2 and 3. Then she went on to explain that $3^{2.5}$ was the same as 3^2 time the square root of 3. Though it seemed that she is relying on properties of exponents, she was able to more fully articulate her response than the students in the control group. Therefore, she exhibited a higher level of understanding (Weber, 2002) than the students in the control group.

Students were also asked if $\log_b x < 0$, then what must be true about x ? No student in the control group was observed answering this question even though the property of negative exponents was written on the board. However, Katie, a student in the treatment group, stated that x had to be less than one. She showed her reasoning by drawing a double number line and showing that if you are less than zero on the bottom of the double number line, then you are less than one on the top of the double number line. Because of the covariational approach, some students in the treatment group were able to transfer their knowledge of the double number line (numerical representation) to the algebraic expression and articulate their answer. Therefore, the observational data showed that the covariational approach allowed students to more easily transfer knowledge and reach a deeper level of conceptual understanding of logarithms.

Though students in the treatment group have been working on problems in mathematics involving pH and $[H^+]$, students in the control group have not been observed voluntarily

transferring their knowledge from chemistry to mathematics. However, lesson 8 was the first time that a student in the control group was observed making a connection to pH. Eddie was trying to evaluate $\log 5 \times 10^{-6}$ in mathematics, and he asked how he could plug this into the calculator. After the instructor showed him, the student stated, “this looks like something we did in chemistry.” Therefore, students in the control group were not observed voluntarily transferring their knowledge from chemistry to mathematics except in this single case.

Quantitative Results

The Logarithms and pH assessment instrument (LPA) was used to compare the control ($n = 17$) and treatment groups ($n = 23$) on their conceptual understanding of logarithms in both a mathematics context and science context. The LPA was designed to measure a student’s conceptual understanding of logarithms by defining conceptual understanding in Algebra as the ability of a student to recognize functional relationships and to interpret different representations of the concept (Panasuk, 2010). A student’s conceptual understanding of logarithms in a mathematics context was measured by comparing the CU factor and the CW and CN sub factors from the mathematics portion of the assessment instrument. In addition, a student’s conceptual understanding of logarithms in a science context was measured by comparing the SN and SW components as well as the overall science score (SO). A Mann-Whitney U test (Table 8) was conducted on each factor and sub-factor to compare the data as well as a power analysis for each statistically significant finding (Table 13)

Table 12

Mann-Whitney U Test on LPA factors

<u>Factor</u>	<u>Control Median</u>	<u>Treatment Median</u>	<u>Mann-Whitney U Statistic</u>	<u>P-value</u>
CW	5	3	122	0.044
CN	2	1	129.5	0.054
CU	4.5	2.5	122	0.045
SW	7	7	184	0.752
SN	3	2	109.5	0.017
SO	4.75	3.5	141	0.135

Note: $p < 0.05$ indicates statistical significance

Table 13

Effect Sizes on Statistically Significant Factors

<u>Factor</u>	<u>Z</u>	<u>Effect Size(η^2)</u>
CW	-2.091	0.11211
CU	-2.013	0.103902
SN	-2.413	0.149297

Among the control and treatment groups on the mathematics portion of the LPA, the CU, CW, and CN factors were analyzed to determine if there exist a statistically significant difference in scores. The Mann-Whitney U test indicated that there was a statistically significant difference on the CU factor among the control group (Mdn = 4.5) and treatment group (Mdn 2.5), $U = 122$, $p = 0.045$ (Figure 28). Therefore, the researcher rejects the null hypothesis that there is no difference in the scores on the CU factor of the LPA. Further, the effect size (η^2) on the CU factor suggest low practical significance ($\eta^2 < 0.20$). The Mann-Whitney U test on the sub-factor CW indicated that there was a statistically significant difference on the CW factor among the control group (Mdn = 5) and the treatment group (Mdn = 3), $U = 122$, $p = .044$ (Figure 29). Therefore, the researcher rejects the null hypothesis that there is no difference in the scores on the CW factor of the LPA. Further, the effect size (η^2) on the CW factor suggest low practical

significance ($\eta^2 < 0.20$). The Mann-Whitney U test on the sub-factor CN indicated that there was not a statistically significant difference on the CW factor among the control group (Mdn = 2) and the treatment group (Mdn = 1), $U = 129.5$, $p = 0.054$ (Figure 30). Therefore, the researcher accepts the null hypothesis that there is no difference in the scores on the CN factor of the LPA. These results indicated that students in the control group outperformed the treatment group on the mathematic portion of the LPA. However, less than 12% of the variability in the ranks on the CU and CW factors can be attributed to the groups that a student participated.

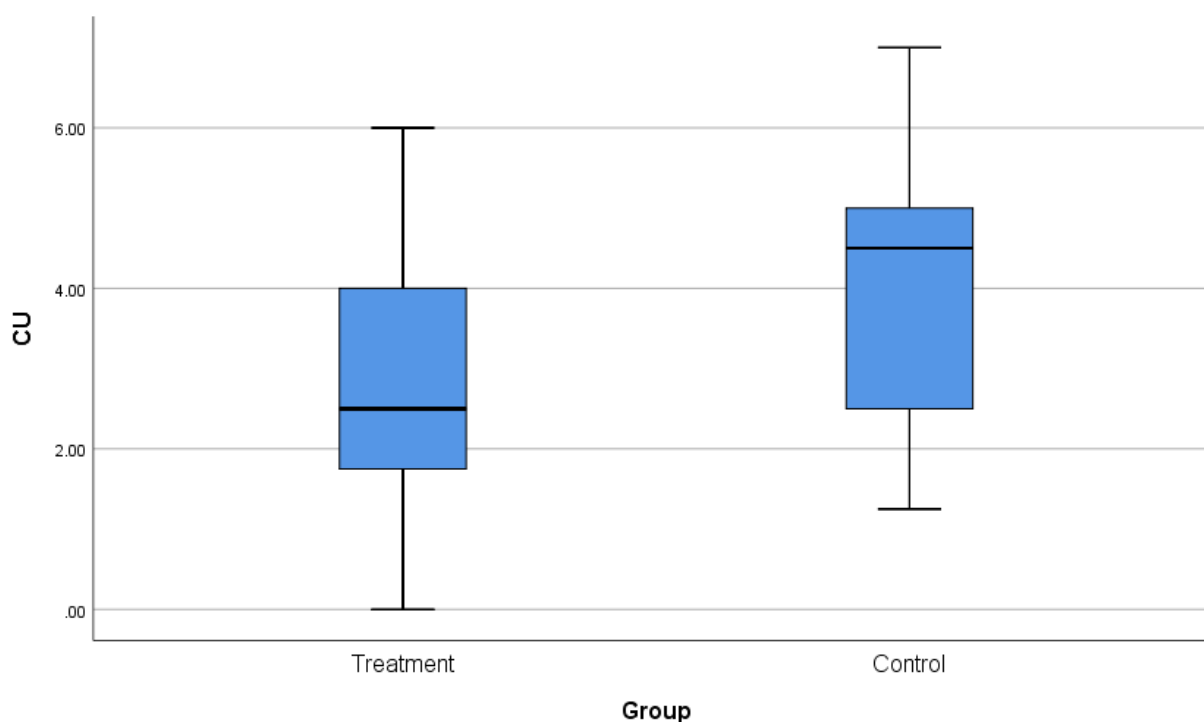


Figure 28: Treatment vs. Control on Overall Score on LPA for Conceptual Understanding of Logarithms in a Mathematics Context (CU)

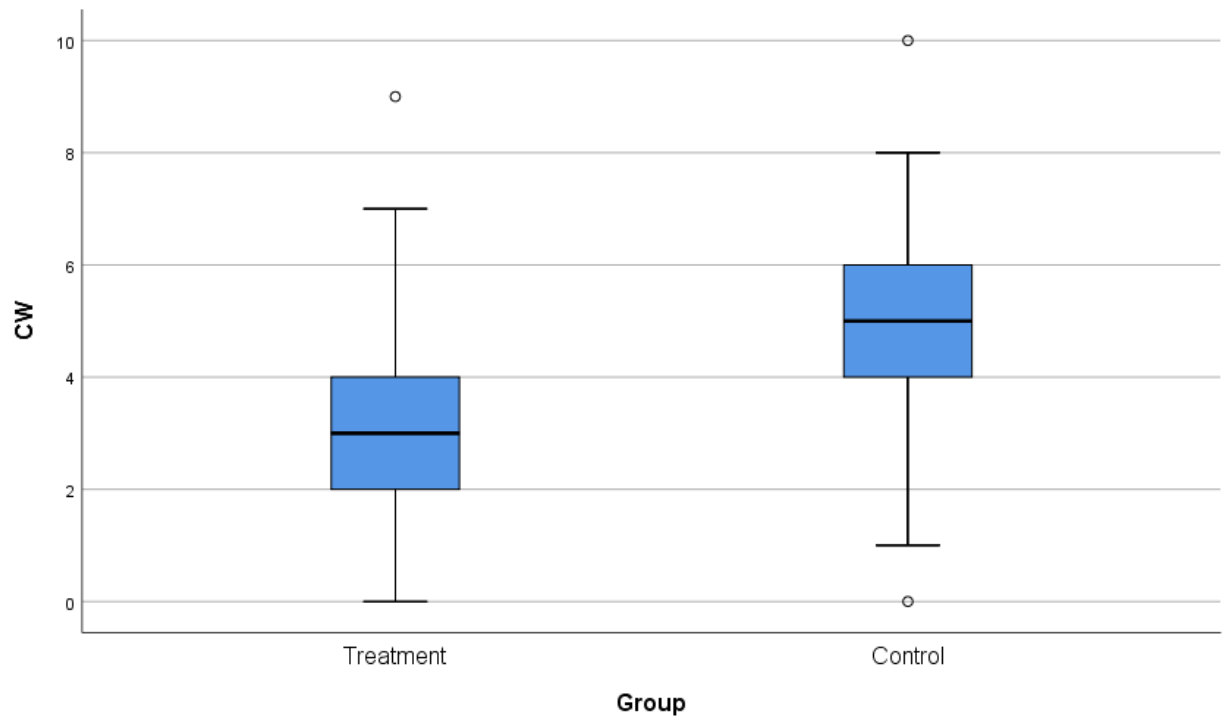


Figure 29: Treatment vs. Control on Overall Score on LPA for conceptual understanding represented through written representations (CW)

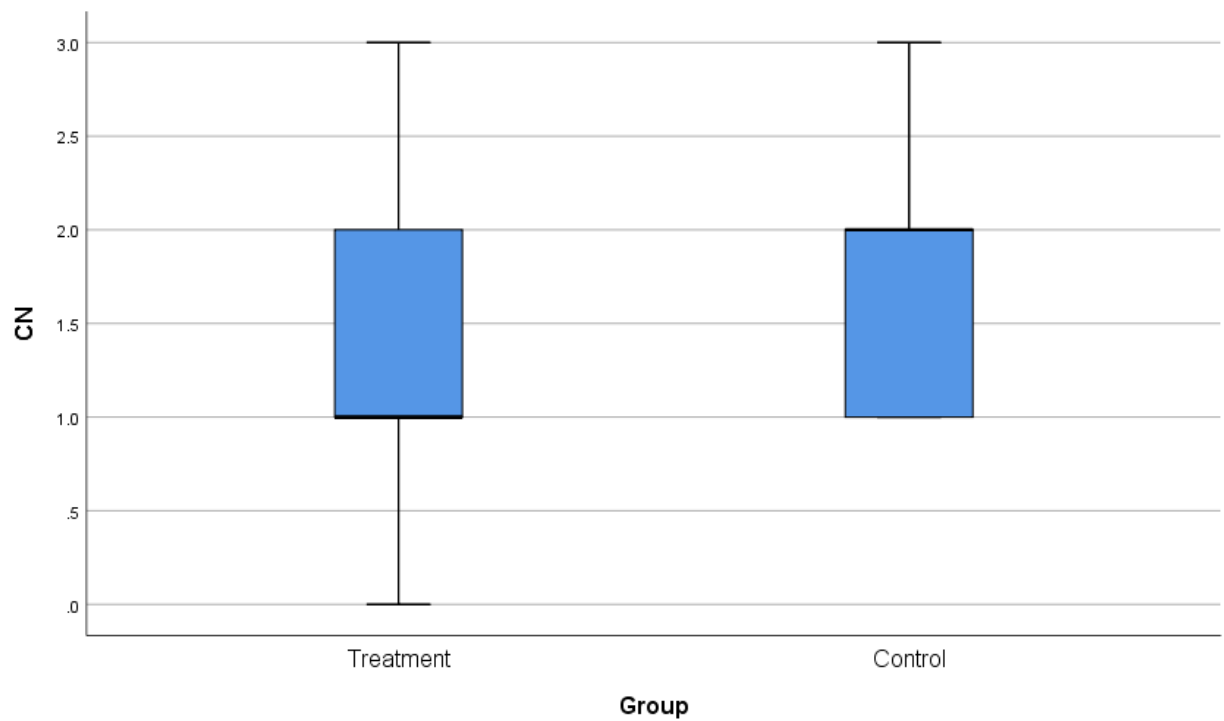


Figure 30: Treatment vs. Control on Overall Score on LPA for conceptual understanding represented through numerical representation (CN).

Among the control and treatment groups on the science portion of the LPA, the SW, SN, and SO factors were analyzed to determine if there exist a statistically significant difference in scores. First, the Mann-Whitney U test indicated that there was not a statistically significant difference on the SW factor among the control group (Mdn = 7) and treatment group (Mdn = 7), $U = 184$, $p = .752$ (Figure 31). Therefore, the researcher accepts the null hypothesis that there is no difference in the scores on the SW factor of the LPA. Second, the Mann-Whitney U test indicated that there was a statistically significant difference on the SN factor among the control group (Mdn = 3) and the treatment (Mdn = 2), $U = 109.5$, $p = 0.017$ (Figure 32). Therefore, the researcher rejects the null hypothesis that there is no difference in the scores on the SN factor of the LPA. Further, the effect size (η^2) on the SN factor suggest low practical significance ($\eta^2 < 0.20$). Last, the Mann-Whitney U test indicated that there was not a statistically significant difference in the scores on the SO factor among the control group (Mdn = 4.75) and the treatment (3.5), $U = 141$, $p = 0.141$ (Figure 33). Therefore, the researcher accepts the null hypothesis that there is no difference in the scores on the SO factor of the LPA. The results from the science portion of the LPA indicated that students in the control and treatment group perform equally well. However, the control group did perform better on the SN factor of the LPA, but only 14% of the variability in the rank scores can be attributed to the group in which a student participated.

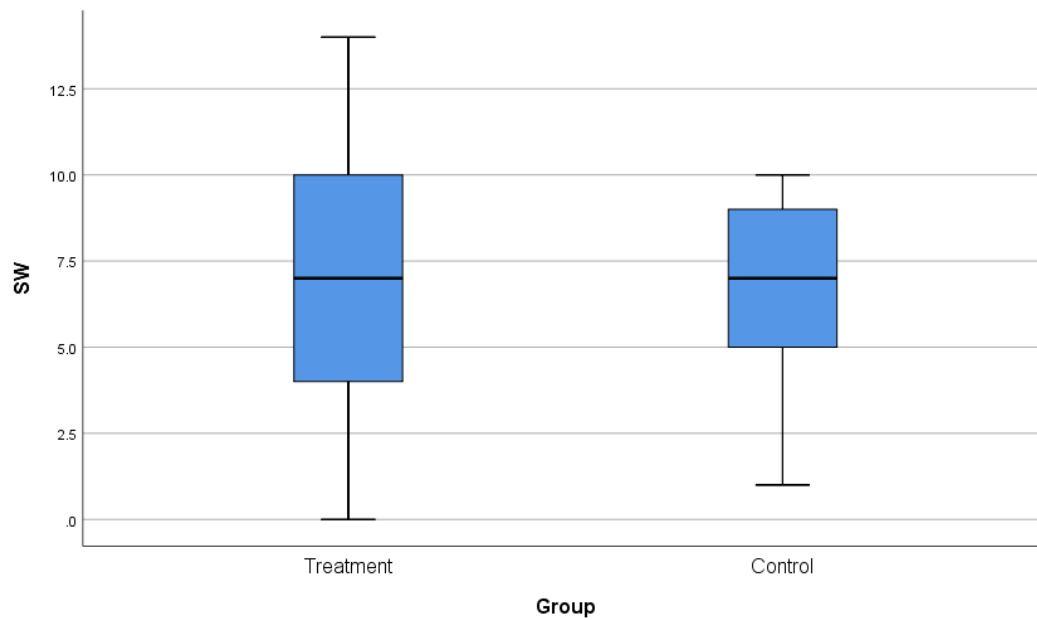


Figure 31: Treatment vs. Control on Overall Score on LPA for conceptual understanding represented through written representations in a science context

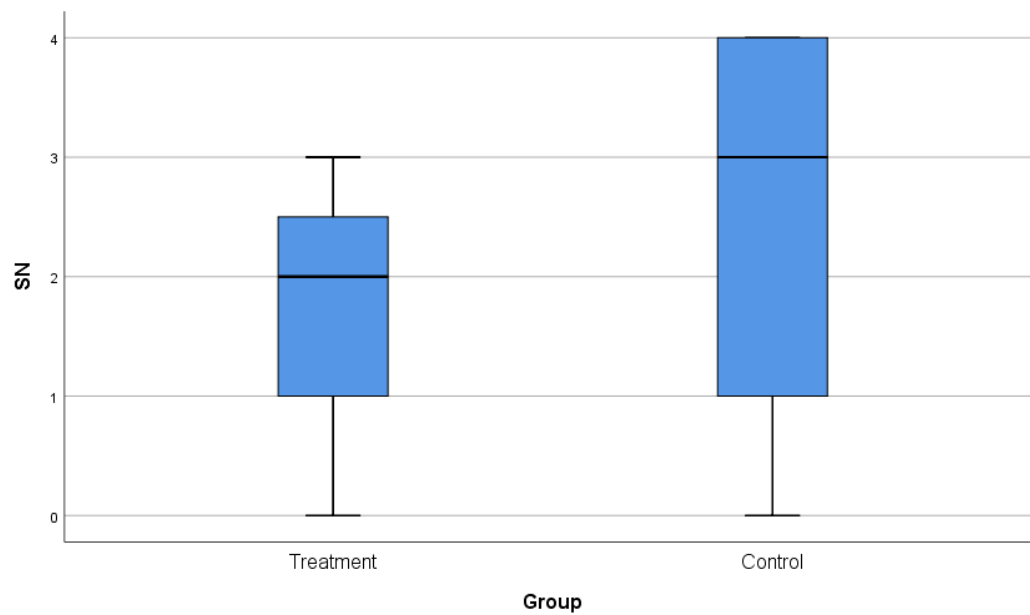


Figure 32: Treatment vs. Control on Overall Score on LPA for conceptual understanding through numerical representations in a science context

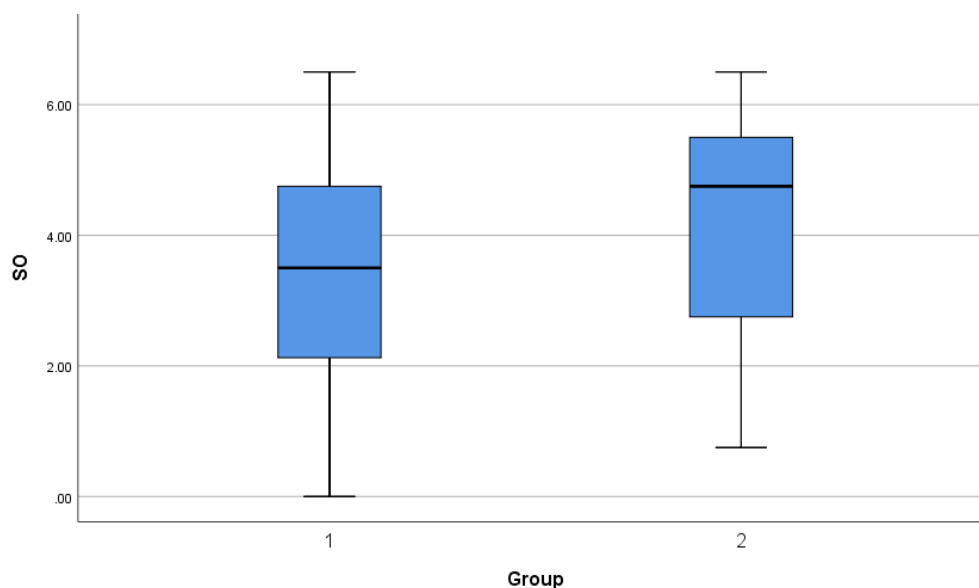


Figure 33: Treatment vs. Control on Overall Score on LPA for Conceptual Understanding of Logarithms in a science context (SO)

Conclusion

The combination of a covariational approach and STEM integration allowed the students in the treatment group to demonstrate a deeper level of conceptual understanding of logarithms and pH. The covariational approach allowed students to use a numerical representation to discover properties, rules, graphical characteristics, and algebraic expressions. They were able to articulate exponents that were not whole numbers, characteristic of exponential and logarithmic graphs such as domain, range, and asymptotes, and the properties of exponents and logarithms. They were able to transfer their knowledge from previous lessons to help them better understand new representations. In addition, most students in the treatment group could use their mathematical knowledge to better understand the concept of pH. Though the data from the LPA does not show that students in the treatment group performed better than the students in the control, the student work and observational data show that the opportunity for a deeper level of conceptual understanding is available in the covariational approach to teaching logarithms.

Chapter 5: Discussion, Limitations, and Future Research

The purpose of this research study was to examine the difference in conceptual understanding of students participating in a traditional classroom setting and students participating in an integrated STEM unit. Conceptual understanding was measured during the lessons qualitatively by students demonstrating through written or verbal comments an ability to represent and understand logarithms according to Weber's (2002) levels of understanding exponential and logarithmic functions and their ability to transfer their knowledge of logarithms to multiple contexts, such as different representations and subject areas. Conceptual understanding was measured quantitatively by the development and implementation of the LPA. The integrated STEM unit connected the concepts of logarithms from Algebra II and pH from chemistry using a covariational approach, while the traditional classroom setting focused strictly on logarithms in mathematics and strictly on pH in chemistry using a correspondence approach. In other words, students participating in an integrated STEM unit were explicitly taught logarithms and pH in both mathematics and science classrooms while students participating in a traditional classroom setting were strictly taught logarithms in a mathematics context and specifically taught pH in a chemistry context.

The lessons in the integrated STEM unit provided an opportunity for students in the treatment group to develop a deeper level of conceptual understanding of logarithms by giving them the ability to communicate their understanding without the use of memorized rules. Weber (2002) explains that for students to reach the deepest level of understanding they would have to be able to articulate their understanding of exponents beyond the use of natural numbers and without the aid of memorized rules. The integrated STEM unit incorporated a covariational

approach to understanding exponents and logarithms and their related rules which prevents students from relying merely on algebraic representations (Confrey & Smith, 1995).

The development of a student's conceptual understanding was evidenced by their use of the covariational approach to communicate concepts during the lessons. While students in the control group were given rules or technology to help them understand logarithms, students in the treatment group developed their knowledge by discovering patterns through examining the rates of change between two related sets.

In lesson one, both groups examined the functions $f(x) = 2^x$ and $f(x) = \log_2 x$. The students in the control group examined these functions by analyzing input values that were natural numbers. Therefore, the development of their conceptual understanding was limited by the correspondence approach and design of the activity. However, students in the treatment group used the base two cards to discover the multiplication and division rules for exponents and logarithms. In addition, students used these cards to extend their knowledge of exponents beyond the natural numbers. They discovered the meaning of the half power and began to examine the meaning of negative exponents.

In lesson two, students in the control group used technology to graph and compare various transformations of base two, three, four, and ten exponential functions. Their understanding of these transformations was limited because of the use of technology. They were not able to articulate why the transformation happened but could only observe that they happened. However, students in the treatment group examined the same functions by using base three, four, and ten cards. They discovered additional rules for logarithms and exponents such as the negative exponent rule, zero exponent rule, product rule for exponents, and the corresponding logarithmic rules. They articulated without memorized rules or facts that they understood that the

base of an exponential and logarithmic function affects the rate of change of the function, and they were able to communicate their understanding of the behavior of the graphs of these functions. For example, several students used the covariational approach to articulate why the graphs have asymptotes. The difference between the control and treatment group continued throughout the unit. Students in the treatment group continued to use the covariational approach to discover and articulate their understanding of graphical, algebraic, and numerical representations.

The lessons in the integrated STEM unit also provided an opportunity for students to apply their knowledge to different representations and contexts to develop their conceptual understanding. Panasuk (2010) and Rittle-Johnson et al. (2001) define conceptual understanding as flexible knowledge that can be applied to multiple types of problems within a context. Within each of the first three lessons of the integrated STEM unit, students in the treatment group used a covariational approach to examine exponential and logarithmic relationships numerically, algebraically, and graphically.

In lesson one, students in both the control and treatment group examined the function $f(x) = 2^x$ numerically by using a table or cards, algebraically by writing the function, and graphically. Unlike the control group, students in the treatment group examined the function beyond the natural numbers to discover properties and characteristics of this function and make connections to multiple representations. Weber (2002) states that to fully understand exponential functions a student must be able to interpret exponents that are negative, rational, or irrational without the use of a given rule. Students should be able to articulate their understanding based on logic and not memorized rules. For example, 22 out of 23 students in the treatment group used the cards (numerical representation) to discover the multiplication and division rules for

both the exponential and logarithmic base two function. 8 out of 23 extended the sequence of cards toward negative infinity to make a connection to the asymptote on the graph.

In lesson two, students in the treatment group examined the logarithmic and exponential functions with a base of three, four, and ten numerically, algebraically, and graphically, while the students in the control group were given the function (the algebraic representation) and asked only to examine the transformation in the graphs using technology. As a result, students in the control group could only explain the differences in the functions by using the graphical representation. However, some students in the treatment group understood the differences in the graphs, because they understood the effects of the base on the rate of change by examining the base 3, 4, and 10 cards. In addition, some students in the treatment group understood the similar characteristics such as the asymptote, y-intercept, and increasing behavior. For example, 20 out of 23 students understood and were able to articulate that extending the double number line to the left would require division of the base of the function. They could see that the top of the double number line would converge to zero and made the connection to the asymptote of the graph through self-discovery (6 students were observed) and through class discussion. The students (10 out of 23) used similar observations by connecting multiple representations to explain the domain and range of the exponential functions.

The lessons in the integrated STEM unit provided the opportunity for students in the treatment group to transfer their knowledge to different context. Honey et al. (2014) defined transfer as the student's ability to apply his or her knowledge and skills from one context to another. Because students do not naturally transfer their knowledge to different contexts such as different STEM disciplines, students need to be explicitly taught different contexts to help them make connections (Honey et al., 2014; Stone et al., 2006). From lesson three until the end of the

integrated STEM unit, students in the treatment group were exposed to and required to apply their knowledge of logarithms in both a mathematics and science context. These students were explicitly being taught how to apply logarithms to pH in the mathematics classroom while learning about pH in the chemistry classroom. The students in the control were only exposed to pH in the chemistry classroom, because pH was not explicitly taught in the mathematics classroom.

The integrated STEM unit provided evidence of transfer because students were able to develop a conceptual understanding of the characteristics and properties of pH. In lesson 3, the students in the treatment group discovered that as pH increases the hydrogen ion concentration $[H^+]$ decreases by using pH cards that were like the cards in the first two lessons. In addition, they were able to deduce that $[H^+]$ could not be zero, when pH is zero $[H^+]$ is equal to one, and the relative difference between the pH of different solutions is a power of ten. They developed this understanding through a covariational approach that they applied to both a mathematics and science context. In the remaining lessons, students in the treatment group were consistently exposed to using logarithms to compare, calculate, and analyze the pH of different solutions. Through this integration, they were able to reach a functional understanding of pH (Park & Choi, 2012). The students in the control group did not show any evidence of transferring their knowledge until the last lesson when a student mentioned that the mathematics they were doing looked similar to something they were doing in chemistry.

In addition to examining the qualitative evidence of conceptual understanding, the researcher sought to quantitatively measure the level of conceptual understanding by developing and implementing the Logarithms and pH assessment (LPA). The LPA consisted of logarithms in a mathematics context and logarithms in a science context sections made of 16 questions each.

The mathematics portion was completed without a calculator while a calculator was allowed by the science teacher. (A calculator was not supposed to be allowed.) A principle component factor analysis (PCA) using SPSS Version 25 was conducted on each section. The factor analysis on the logarithms in a mathematics context returned one factor which was called conceptual understanding (CU). When comparing the two groups using a Mann Whitney U test, there was a statistically significant difference in favor of the control group. In addition, the factor analysis of the science section returned two factors called SW (Science writing) and SN (science numerical). When comparing the two groups using a Mann Whitney U test, there was a statistically significant difference in favor of the control group for SN and not a statistically significant difference for the SW factor.

Though these results are not in favor of the treatment, they can be explained. First, Weber (2002) states that to reach the Generalization level of understanding of exponents and logarithms, a student must not rely on given rules to articulate the meaning of exponents that are not natural numbers. However, the students in the control group built their knowledge of exponents and logarithms from given rules. This suggests that a student can articulate complex ideas of logarithms by using given rules. As a result, the students in the control group can perform well on an assessment designed to measure a student's conceptual understanding through multiple representations that does not require students to articulate the reasons for their answers beyond the use of a given rule or fact. Second, the study was conducted using on-level Algebra II students in which 50% of the students in both groups were considered "Developing learners" (GSE, 2015) according to their Algebra I EOC scores. Honey et al. (2012) states that "integration can also impede learning because it can place excessive demands on resource-limited cognitive processes, such as attention and working memory" (p. 4). Therefore, the

treatment group's success could have been limited by their cognitive ability. Last, calculators were allowed to be used on the science portion of the LPA. Leopold and Edgar (2008) state that calculators can hide a student's deficiencies in their understanding, and therefore allow the student to demonstrate high-level abilities. Consequently, the control group's success on the SN factor of the LPA could be contributed to the use of a calculator. In addition, the treatment group was not allowed to use calculators during their unit, and therefore, they were not shown how to evaluate logarithms in a calculator.

In conclusion, the explicit integration of logarithms and pH did allow the students the opportunity to reach a higher level of conceptual understanding of both logarithms and pH as shown by the qualitative data. Students in the treatment group were more equipped to reach a higher level of conceptual understanding because their knowledge of logarithms and pH was not based on given rules but on observations made through covariational reasoning. However, the level at which a student reaches could be limited by their cognitive ability to use their knowledge in multiple contexts as evidenced by the LPA.

Implications

The researcher's purpose was to examine the difference in conceptual understanding of logarithms and pH among students participating in a traditional classroom setting and students participating in an integrated STEM unit. As measured by the LPA, students in the integrated STEM unit on average reached a level of conceptual understanding of logarithms and pH that was equivalent to or lower than the students in a non-integrated setting. However, the qualitative data revealed that the lessons in the integrated STEM unit gave students the opportunity to reach a higher level of understanding during the unit, because their understanding is based on observations discovered using covariational reasoning. They made comments that revealed a

generalized level of thinking. In addition, students in the integrated STEM unit made connections between logarithms and pH that were not evident in the non-integrated setting. Therefore, this study reveals several important factors that impact mathematics and science educators and teacher educators.

First, mathematics and science educators that focus on conceptual understanding should not solely rely on quantitative data to measure a student's success. Weber (2002) stated that for a student to reach a generalized knowledge of exponential or logarithmic functions they need to be able to articulate their understanding without the use of memorized rules. And though students may be able to articulate complex ideas of exponential and logarithmic functions, if these ideas are based on memorized rules, they have not reached a high level of conceptual understanding. As evidenced by the LPA, students in the control group that focused on rules and definitions performed slightly better. However, the students relied on memorized rules and definitions that were shared by the teacher to analyze and solve problems containing logarithms and pH while the students in the treatment group relied on covariational reasoning to develop rules and solve problems. Panasuk (2010) described a student's reliance on memorized rules and following procedures shown by the teachers as process conception and that these students may not have developed conceptual understanding. In addition, the qualitative data showed that students in the control group had more difficulties than the treatment group at extending their knowledge of logarithms beyond the natural numbers without the use of rules. Therefore, students in the control group gained knowledge that limits their ability to further develop their conceptual understanding while the students in the treatment group are equipped to develop a deeper understanding of logarithms and pH.

Second, mathematics and science educator should consider a student's ability to articulate complex ideas without given rules or procedures when assessing conceptual understanding. The LPA was designed to measure conceptual understanding of logarithms and pH by defining conceptual understanding as flexible knowledge that can be applied to multiple types of problems within a context (Panasuk, 2010; Rittle-Johnson et al., 2001). The researcher designed the LPA to measure a student's understanding of logarithms by providing problems that required a numerical, algebraic, and graphical understanding of logarithms. However, Weber (2002) stated that for students to reach the Generalization level of understanding they need to be able to articulate their understanding without the use of memorized rules given by the teacher. Therefore, the LPA could be improved by changing some of the questions from multiple choice to short answer, and by adding more questions that require students to articulate their reasoning for a response.

Third, mathematics and science educators should look for opportunities to integrate the subjects to help students make connections and transfer knowledge among related concepts in different content areas. Students in the treatment group benefited from the integration of logarithms and pH, because they were able to use mathematics to better understand pH and its relationship to hydrogen ion concentration. In addition, students' knowledge of logarithms was impacted, because they were forced to apply their knowledge to a real-world context throughout the unit and not just in one lesson. As evidenced by the qualitative data, students in the control group did not show that they made any connection between logarithms and pH even though they were taught simultaneously. However, students that participated in the integrated STEM lesson regularly encountered situations in which their knowledge from both mathematics and chemistry

was required. This allowed for students to apply their knowledge and examine how mathematics and science are connected with the guidance of content experts.

Fourth, teacher educators should provide pre-service and in-service teachers with experience integrating different content areas. Potgieter et al. (2008) stated that students in chemistry have difficulties with topics that are mathematically intensive because of the lack of conceptual understanding of the mathematical content involved. Burghardt et al. (2010) stated that making connections between STEM disciplines can help broaden a student's understanding of concepts. Therefore, educators need to be equipped with knowledge that is beyond their specific content area and how to integrate concepts from other STEM disciplines into their content area. During the study, students in the treatment group benefited from the integration of mathematics and chemistry, because the teachers in the study were provided a way to teach both logarithms and pH using similar techniques. Covariational reasoning provided a common link that helped the teachers in the study make a more seamless connection between the content areas.

Last, mathematics and science educators should consider the "Base 2 Card", the "Different Bases", and the "pH and logarithms" tasks as well as covariational reasoning when designing a unit to teach logarithms or pH. For mathematics educators, the "Base 2 Card" and the "Different Bases" tasks using covariational reasoning helped students to build a foundation for the properties of logarithms that were not based on given rules. In addition, these activities as well as the "pH and logarithms" task helped students understand the meaning of the base of an exponential and logarithmic function, understand asymptotes and other graphical characteristics by making connections between multiple representations, and understand how to use logarithms in other content areas. Mathematics educator should also consider using covariational reasoning, because it allowed the students to make better sense of negative, rational, and irrational

exponents. For science educators, the “pH and logarithms” task helped students understand the functional relationship between pH and hydrogen ion concentration. Students were able to discover that pH increases as concentration decreases, the relative difference between pH values, and that pH is not limited to the 0 to 14 range.

In conclusion, mathematics and science educators have opportunities to integrate STEM subjects to equip students to develop a deeper level of conceptual understanding of each content area involved and to transfer that understanding from one content area to another. However, integration of the STEM subjects should not be made implicitly. Students should not be expected to make these connection without the guidance of the content expert. Therefore, mathematics and science educators should explicitly integrate different content areas to equip students for achieving deeper levels of conceptual understanding and transferring knowledge to other content areas. In addition, teacher educators should equip pre-service and in-service teachers with experience and knowledge to integrate content areas, so they are better able to create and implement integrated STEM lessons.

Limitations

The study measures the effects of an integrated STEM unit that uses covariational reasoning on the level of conceptual understanding of logarithms and pH. The study measures conceptual understanding using both quantitative and qualitative data and gives insight for educators and teacher educators when developing integrated STEM lessons. However, it is important to understand the limitations of the study.

First, the sample size and the sample should be considered when attempting to generalize the results to different student populations and understanding the limitation of the data collected from the LPA. The researcher was limited to the amount and type of students available for the

study. The researcher was restricted by the class schedules of the students, because the school size did not allow for much flexibility when creating student schedules. Therefore, the lack of a probability sampling technique limits the generalization of the study results to other populations. In addition, the demographics of the sample available for the study was limited to a population of students in an on-level algebra II class. Honey et al. (2012) states that “integration can also impede learning because it can place excessive demands on resource-limited cognitive processes, such as attention and working memory” (p. 4). Therefore, motivation and the lack of working memory could be confounding variables. In addition, the quality of the LPA in assessing the level of conceptual understanding of logarithms and pH is limited. The use of a convenience sampling technique and the sample size limits the quality of the data collected. As mentioned, the researcher could only use the students that were available because of class schedules and school size. In addition, a factor analysis was conducted on the LPA and three factors were extracted. However, Pallant (2005) states that factors extracted from a small sample do not generalize as well.

Second, the qualitative data shows that the students in the treatment group could reach a higher level of understanding through the integrated STEM unit and covariational reasoning. It should be noted that the opportunity for a higher level of understanding does not mean that the treatment group on average reached a higher level of understanding. The qualitative data collected was based on student work and observations made during the lesson. However, the researcher was the only instructor in the room and data were not collected from all individuals in each group during each lesson. Therefore, the study could be improved by including more qualitative data in the process.

Third, the counts from student work could be inflated based on the class discussion at the end of each lesson. In both the control and treatment groups, the instructor led class discussions at the end of each lesson, so students could discuss and be exposed to the goals of each lesson. Students were able to share answers and thoughts that could have been written down by other students before the students' work was collected. Therefore, some student work may show a deeper level of understanding than the students reached on their own.

Fourth, the control group did not have the opportunity to show a deep level of understanding because of the teaching approach, and the treatment group's procedural skills are limited. The treatment group was taught using covariational reasoning and given lessons that required the use of covariational reasoning, while the control group was taught a correspondence approach based on given rules. Weber's (2002) levels of conceptual understanding requires students to have the ability to articulate the meaning of rational, negative, and irrational exponents without the use of given rules to reach the deepest level of understanding. Therefore, the students that were taught using covariational reasoning were able to show a generalized level of understanding that the control group could not. The control group was taught using a correspondence approach and was given the properties of logarithms and exponents to articulate complex ideas of logarithms and exponents. However, the control group did have more practice with the procedural skills for solving exponential and logarithmic functions. The integrated STEM lessons in which the treatment group participated did not emphasize procedural skills and taught them more implicitly than explicitly. Students in the treatment group were required to use a conceptual approach to solving logarithmic problems while students in the control group were shown the procedures to solve logarithmic and exponential problems.

Fifth, the qualitative data were limited to the mathematics classroom. Observational data and student work were only collected in the mathematics classroom. Students in both groups were not observed transferring knowledge of logarithms to pH in the chemistry setting. Therefore, the conclusions about transferring knowledge between mathematics and science is limited to students transferring their knowledge between logarithms and pH in the mathematics setting.

Last, the LPA was designed to measure conceptual understanding of logarithms and pH by defining conceptual understanding as flexible knowledge that can be applied to multiple types of problems within a context (Panasuk, 2010; Rittle-Johnson et al., 2001). The researcher designed the LPA to measure a student's understanding of logarithms by providing problems that required a numerical, algebraic, and graphical understanding of logarithms. However, Weber (2002) states that for students to reach the Generalization level of understanding they need to be able to articulate their understanding without the use of memorized rules given by the teacher. Therefore, the LPA could be improved by changing some of the questions from multiple choice to short answer, and by adding more questions that require students to articulate their reasoning for a response.

Future Research

As a result, further studies on developing conceptual understanding through integrated STEM lessons can be improve by considering the limitations of this study. First, future research needs to be conducted on multiple types of student populations such as upper-level students or on larger more randomized samples to control for possible confounding variables such as motivation and working memory. For example, students pursuing STEM degrees may benefit from these lessons. Future studies could use the “pH and Logarithms” task in an introductory

college Chemistry class to investigate the benefits of teaching pH conceptually and to investigate if students can better use their knowledge of logarithms in the chemistry context with the covariational approach. Second, future research can focus this type of study on the science classroom, so the integrated STEM unit and the covariational approach can be observed in a different setting. Students in this study were observe transferring their knowledge between logarithms and pH in the mathematics setting, but they were not observed in the chemistry setting. Therefore, the researcher does not know if the student in either the control or treatment group voluntarily transferred their knowledge in the chemistry setting. Last, quantitative instruments need to be designed or improved to account for a broader definition of conceptual understanding. The LPA limited the definition of conceptual understanding to flexible knowledge that could be applied to multiple types of problems within a context. However, Weber (2002) suggests that students can answer multiple types of questions within a context without having true conceptual understanding. He suggests that students need to be able to articulate their responses without relying on given rules. Therefore, future research could focus on developing assessment instruments that measure conceptual understanding by including short answer or essay type problems that require students to articulate the reason for a particular response. These types of instruments will allow for a more thorough measurement of a student's conceptual understanding.

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Appendix A

Park & Choi (2012) Interview Protocol

1. If you have heard of pH values, explain which areas/subjects the terms are used in.
 2. Define or explain what the pH value is.
 3. Explain what a typical range of pH values is (What is the value at both ends of the scale?)
 4. Explain the purpose of pH value or how it is used. (You may explain it by giving specific examples or descriptions.)
 5. In order to use pH values for the purpose as described in item 4, explain or illustrate how you can get pH values. (You may explain it by giving specific examples or descriptions.)
 6. Assuming that there are four pH values of 1, 2, 3, 4, among them, explain what the relative difference between pH 1 and 3 is. (i.e. What does the value difference mean? How much are they different? How are they different, etc.?)
 7. Based on your explanation to question 6, represent the values on a line below and then, explain the meaning of the space between the values and the relative difference between values pH 1 and 3.
8. HCl solutions in seven different molarities were obtained from a chemistry laboratory.

HCl (hydrochloric acid) aqueous solution: 0.1, 0.02, 0.001, 0.0002, 0.00002, 0.000001, 0.0000002 M.



Display the relationship between molarities of HCl and pH values in the Cartesian coordinates below and then explain the meaning of graphical representation in terms of the relationship between the two values.

9. After reviewing the two responses to questions 7 and 8, explain any common factors or differences between the two representations with respect to explaining the relative differences of pH values

Observation Protocol

[illegible]

Appendix C

Logarithms and pH Assessment Instrument

1. Define or explain a logarithm.
2. Explain the meaning of the following expression using complete sentences.

$$\text{Log}_2 5$$

3. Is it possible to evaluate $\log_2(-5)$? Why or Why not?
4. Use the following sentence to explain the range of $f(x) = \log_3 x$. (Make sure you fill in the blanks complete the sentence.)

The range of the function is _____ (minimum) to _____ (maximum),
because ...

5. Which of the following choices is the largest?
 - a. $\log_2 20$
 - b. $\log_4 45$
 - c. $\log_9 80$
 - d. $\log 1000$
6. Use the properties of Logarithms to find the exact value of the expression

$$\log_2 14 - \log_2 7$$

- a. 1
 - b. 2
 - c. 7
 - d. 14
7. Using the answer choices below, between which two whole numbers is $\log_2 11.53$?
 - a. 0 and 1

- b. 3 and 4
 - c. 5 and 6
 - d. 9 and 10
8. Let $\log_3 4 = 1.3$ and $\log_3 20 = 2.7$, solve $\log_3 5$
- a. 1.4
 - b. 2.1
 - c. 3.5
 - d. 4.0
9. If $\log_b 10 = 3.32$, then approximate the value of x in the following equation.

$$\text{Log}_b x = 6.64$$

- a. 12
 - b. 20
 - c. 100
 - d. 44
10. If $\log_5 xy < 0$, which of the following must be true.
- a. xy is less than zero
 - b. xy is less than one
 - c. xy is greater than one
 - d. xy is greater than zero
11. Rewrite the following logarithmic expression in exponential form.

$$\text{Log}_5 6.3 = x$$

- a. $5^{6.3} = x$
- b. $5^x = 6.3$
- c. $x^{6.3} = 5$
- d. $x^5 = 6.3$

12. Solve for x in the following equation

$$\log_5(3x + 1) = 2$$

- a. $\frac{1}{3}$
- b. 3
- c. 2
- d. 8

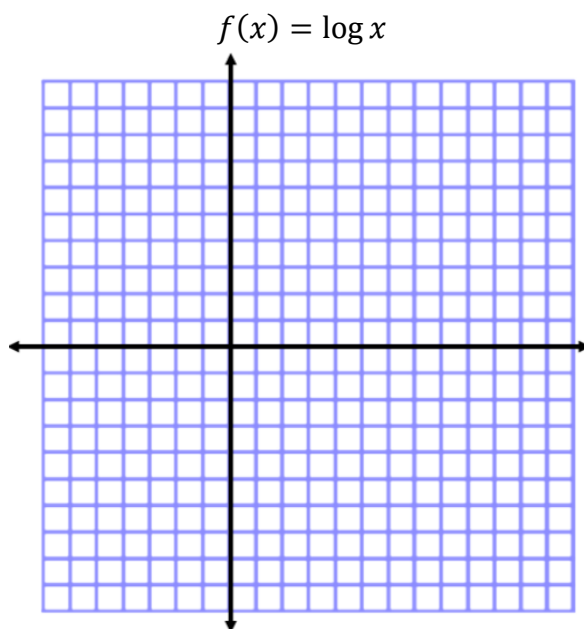
13. Approximately what number is indicated by the arrow?



- a. 2.0×10^5
- b. 3.0×10^5
- c. 5.0×10^5
- d. 5.0×10^6
- e.

14. Using the seven different values listed below for x , sketch a graph of the following function.

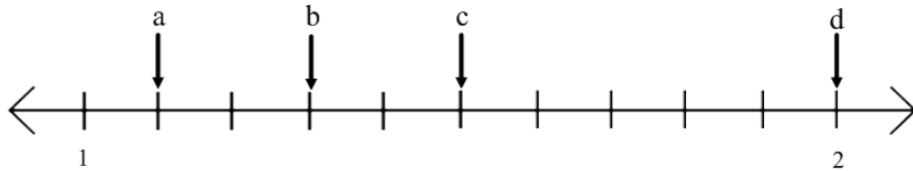
X	.001	0.1	10	1000	10023	1000000	10^8
y							



15. What is the domain, asymptote, x-intercept, and y-intercept of the following function?

$$f(x) = \log_4(x - 1)$$

16. Which arrow indicates the location of $\log 20$ on the number line



Science Context Questions

1. Define or explain the pH value.
2. Explain the meaning in context of pH for the following using complete sentences.

$$-\log(1 \times 10^{-7})$$

3. Is it possible to have hydrogen ion concentration $[H^+]$ of 0? Why or Why not?
4. Explain the range of pH values? (You can explain by giving specific examples or descriptions)
5. Which solution has the most concentration of hydrogenions?
 - a. pH of 4
 - b. pH of 7
 - c. pH of 13
 - d. pH of 12.3
6. What is the relative difference between the hydrogen ion concentration of solutions with a pH of 1 and a pH of 3?
 - a. .099
 - b. 2
 - c. 20
 - d. 100
7. Drinking water has a $[H^+]$ that ranges from 3.16×10^{-9} M to 10^{-6} M. Determine the approximate range for the pH of drinking water.
 - a. 6 to 9
 - b. 6 to 8
 - c. 6 to 8.5
 - d. 6 to 9.5

8. If the pH of orange juice is 3.2, then it has a hydrogen ion concentration of $6.3 \times 10^{-4}\text{M}$. What is the hydrogen ion concentration of a solution with a pH of 5.2?
 - a. $8.3 \times 10^{-4} \text{ M}$
 - b. $6.3 \times 10^{-5} \text{ M}$
 - c. $8.3 \times 10^{-5} \text{ M}$
 - d. $6.3 \times 10^{-6} \text{ M}$

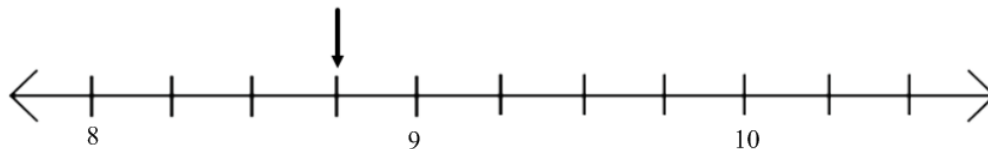
9. What is the molar concentration of H^+ in an aqueous solution with a pH of 4.3?
 - a. $2.3 \times 10^{-5} \text{ M}$
 - b. $5.0 \times 10^{-5}\text{M}$
 - c. $2.3 \times 10^{-4} \text{ M}$
 - d. $5.0 \times 10^{-4}\text{M}$

10. If the pH of an unknown solution is less than 5, then which of the following must be true?
 - a. $[\text{H}^+]$ *less than* $1 \times 10^{-4}\text{M}$
 - b. $[\text{H}^+]$ *less than* $1 \times 10^{-5}\text{M}$
 - c. $[\text{H}^+]$ *greater than* $1 \times 10^{-6}\text{M}$
 - d. $[\text{H}^+]$ *greater than* $1 \times 10^{-5}\text{M}$

11. If the pH of bleach is 13.5, then what is the hydrogen ion concentration?
 - a. $10^{13.5} \text{ M}$
 - b. $10^{1.35}\text{M}$
 - c. $10^{-1.35}\text{M}$
 - d. $10^{-13.5}\text{M}$

12. If the pH of sea water is 8 and the difference of pH between coffee and sea water is 3 (pH of sea water minus pH of coffee), what is the hydrogen ion concentration of coffee.
 - a. -5 M
 - b. 10^{-5} M
 - c. 5 M
 - d. 10^5 M

13. Approximately what is the hydrogen ion concentration of the indicated arrow on the pH scale?



- a. $1.7 \times 10^{-9} \text{ M}$
- b. $3.1 \times 10^{-9} \text{ M}$
- c. $8.75 \times 10^{-9} \text{ M}$
- d. $2 \times 10^{-10} \text{ M}$

14. HCl solutions in seven different molarities were obtained from a chemistry laboratory.

HCl (hydrochloric acid) aqueous solution: 0.1, 0.02, 0.001, 0.0002, 0.00002, 0.000001, 0.0000002 M.

Calculate and Display the relationship between molarities of HCl and pH values in the Cartesian coordinates below.

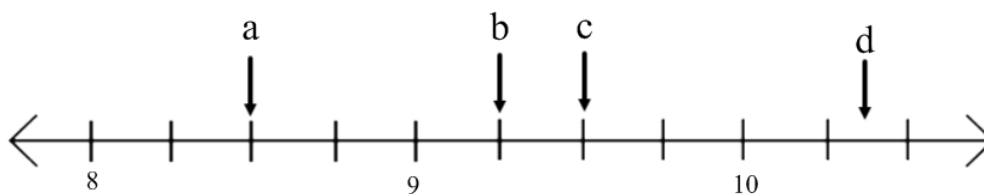


15. Provide an interval for hydrogen ion concentration and pH value and explain the interval.

Hydrogen ion:

pH value:

16. Baking Soda has a hydrogen ion concentration of $3.16 \times 10^{-10}M$. Which arrow indicates the pH value of baking soda?



Appendix D

Base 2 Card Game Task

Time: two 50-minute periods or one 90minute block

Essential Question: What type of function relates an arithmetic sequence to a geometric sequence?

Learning Objective:

- Students will understand the relationship between the domain and range of a function with a base of 2.
- Students will understand the relationship between multiplying in a geometric sequence and adding in an arithmetic sequence (subtracting and dividing will be discussed as well)
- Students will understand the relationship between multiplying in an arithmetic sequence and repetitive multiplying in a geometric sequence.
- Students will be able to identify the characteristics of the graph of a base 2 function.
- Students will understand the meaning of a negative exponent.

Standards:

- MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology. (Limit to exponential and logarithmic functions.)
- MGSE9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- MGSE9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Limit to exponential and logarithmic functions.)
- MGSE9-12.F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Materials:

- Base 2 Card Game students sheet
- Base 2 Cards (cut out and ready to use)

Differentiated Instruction

- In the students-centered task, the teacher will monitor student progress and ask probing questions to help each group on-task and progressing through the activity

Assessment Strategies:

- Group interviews
- Monitoring activity

Lesson Preparation:

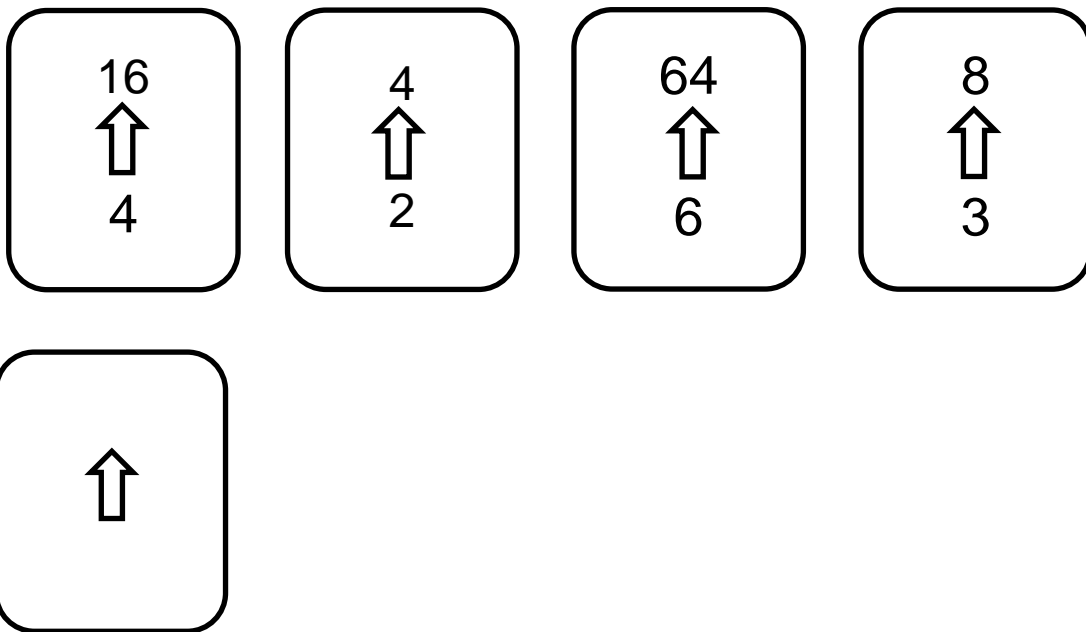
1. Cut out Base 2 Cards. Make enough sets of cards to allow groups of 2 or 3 to have a set.
2. Copy Base Card Game Students sheet. Provide a copy for each student.
3. Prepare a SmartBoard or PowerPoint file with the discussion question at the end of the task.

Base 2 Card Game Task

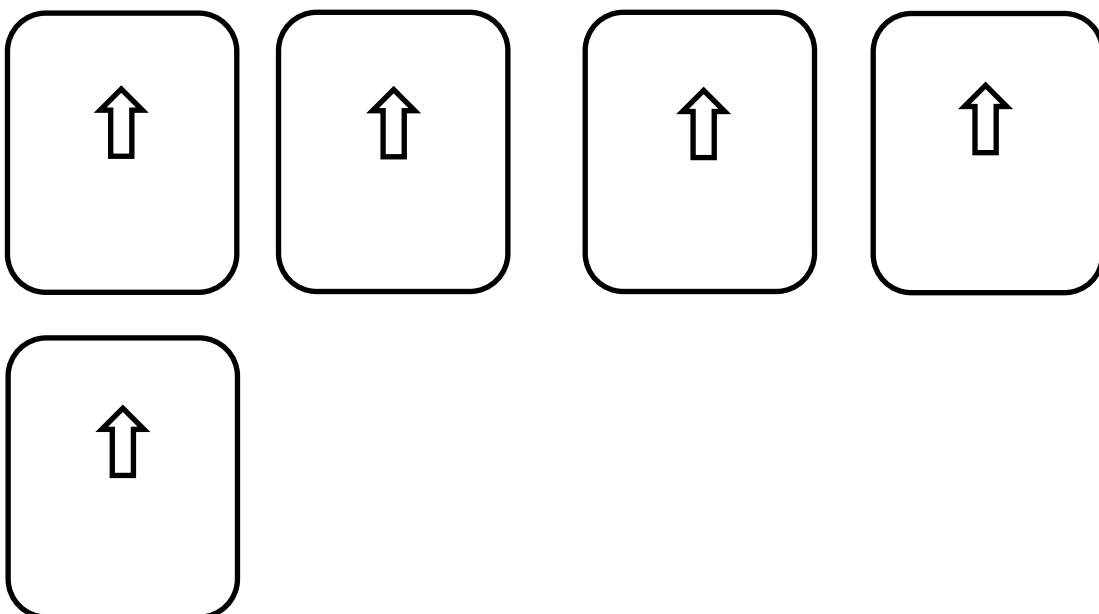
The following task is adapted from Ferrari-Escolá, Martínez-Sierra, and Méndez-Guevara (2016).

1. Making more cards

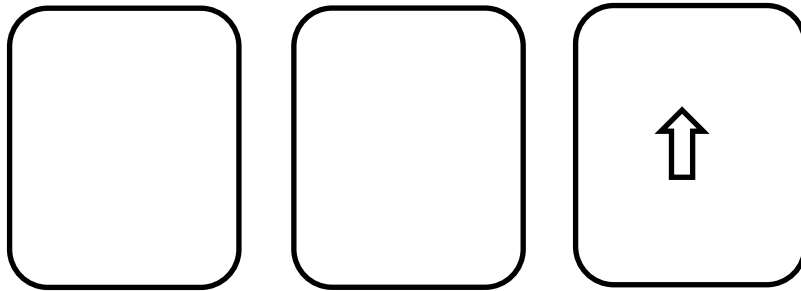
There is a card game in which some of the cards have been lost. We need to rebuild the game and discover the rules that govern it. We have found some of the cards from the game. Could you find the one that is missing and build it?



2. The game requires at least 10 cards to play. Construct more cards and explain the rule that you must follow to build more cards.



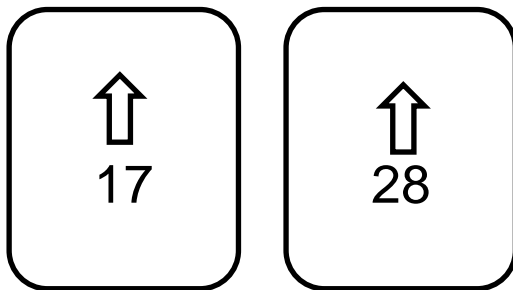
3. It is possible to multiply and divide using these cards. How do you think they can be used for this purpose? (You might have to build more cards when you multiply or dividing.)



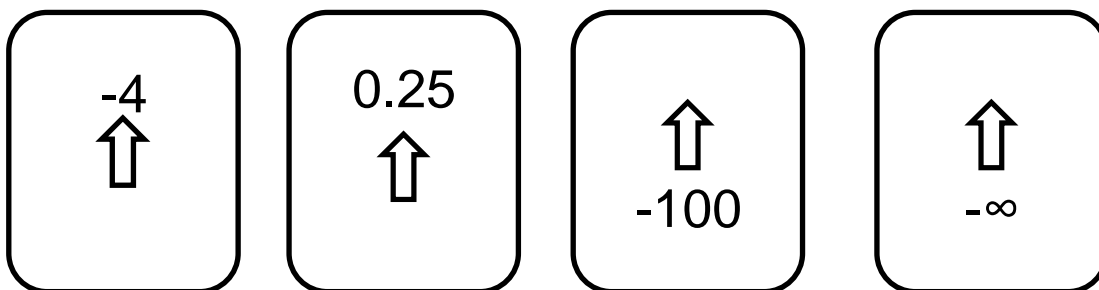
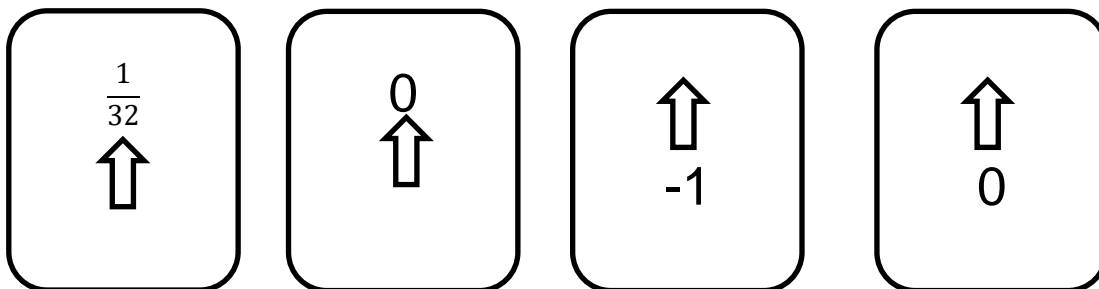
Explain the multiplication rule that you have found.

Explain the division rule that you have found.

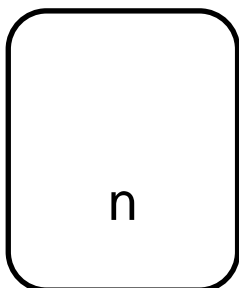
4. Using the cards created so far, finish building the ones below and explain what you did.



5. Which of the following cards could belong to the game according to the established rules? If the card belongs, build the card and explain how it was built, or if you reject the card, explain your reasoning. (It might be helpful to order the existing cards and try to fit the cards below in your ordering)



6. How could we build any card? That is, what is the general form of the card? What would the other part of the card be if n was given?

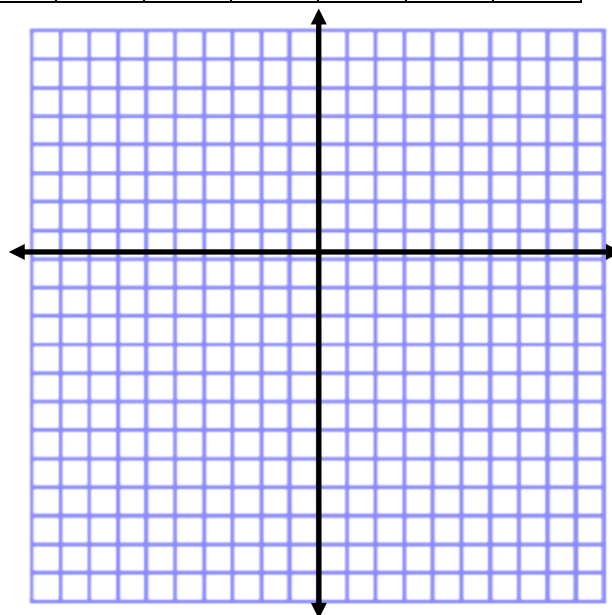


$$f(n) =$$

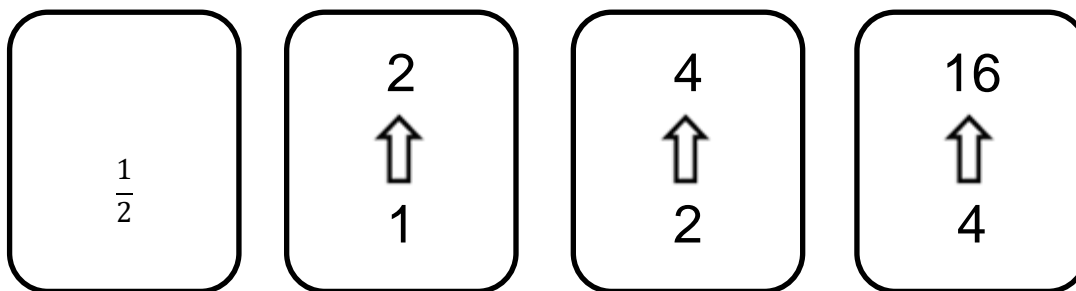
7. Use the cards that you have created to complete the table and plot the points on the coordinate plane. In addition, describe the graph by finding the domain, range, intercepts, end-behavior, asymptote, and intervals of increase and decrease.

X	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$												

- Domain:
- Range:
- Intercepts:
- End-behavior:
- Asymptote:
- Interval of increase:
- Interval of decrease:



8. Now consider the following cards and use the graph above to help identify (estimate) the corresponding value.

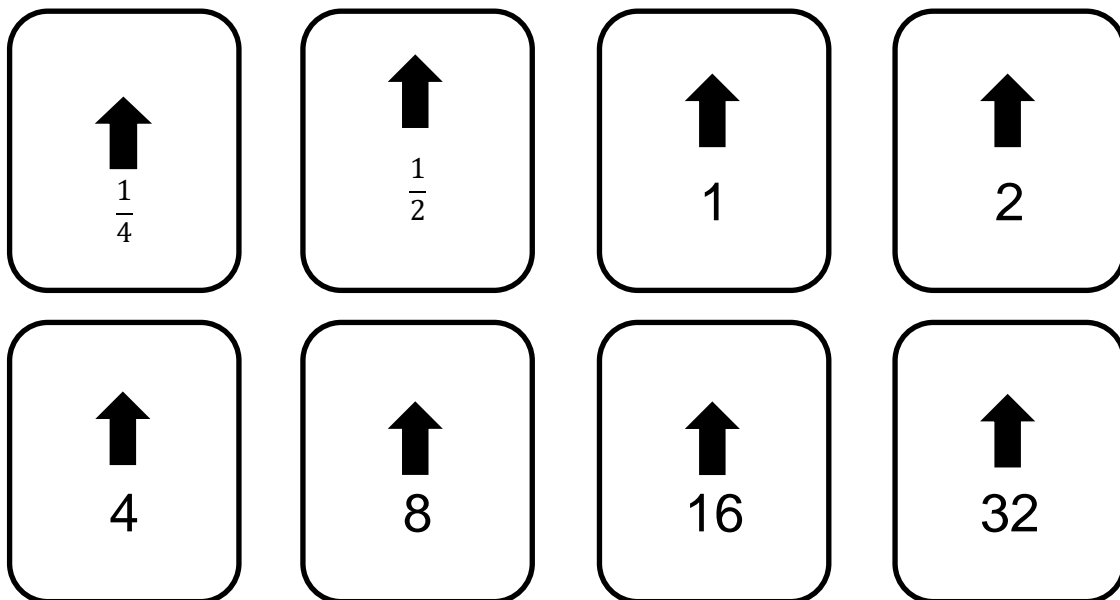


What is happening to the bottom row of numbers as you move from right to left?

What is happening to the top row of numbers as you move from right to left?

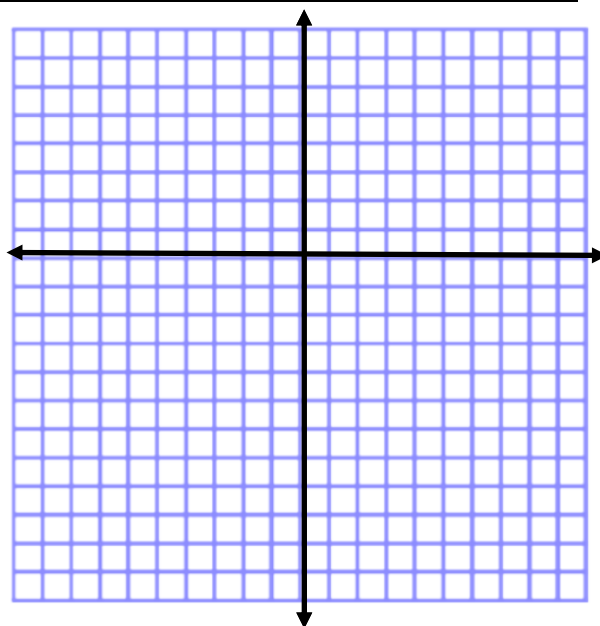
If $n = \frac{1}{2}$, then what math operation are you using to calculate the corresponding number?

9. What happens if the game cards are inverted? For example, instead of $2 \rightarrow 4$, let $4 \rightarrow 2$. Construct the cards below. Then make a table, graph the new function, and identify the indicated characteristics. Use the bottom row as the x-values and the top row as the y-values.



x												
$f(x)$												

- Domain:
- Range:
- Intercepts:
- End-behavior:
- Asymptote:
- Interval of increase:
- Interval of decrease:



Class Discussion

- What is the rule for multiplication?
- What are the characteristics of the first graph? (Make sure you have a completed graph shown to the class)
- What are the characteristics of the second graph? (Make sure you have a completed graph shown to the class)
- What function did you create in number 6?

Appendix E

Different Bases Task: Discovering Properties

Essential question: What is a logarithm?

Learning Objectives:

- Students will use the knowledge on the Base Two Card Game Task to and the current task to develop properties of exponents and logarithms

Standards:

- MGSE9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Limit to exponential and logarithmic functions.)
- MGSE9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (Limit to exponential and logarithmic functions.)

Materials

- Different Bases Task student sheet
- Different Bases Task cards (cut out and ready to use)

Differentiated Instruction

- In the students-centered task, the teacher will monitor student progress and ask probing questions to help each group on-task and progressing through the activity

Assessment Strategies

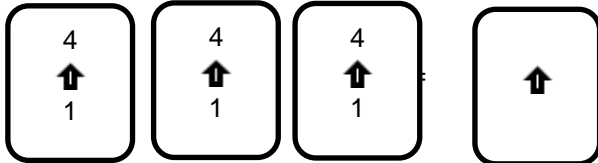
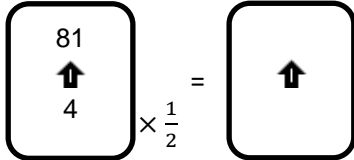
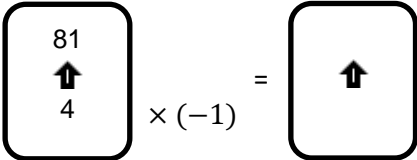
- Group interviews
- Monitoring activity

Lesson Preparation:

4. Cut out Different Bases task cards. Make enough sets of cards to allow groups of 2 or 3 to have a set.
5. Copy Different bases task Students sheet. Provide a copy for each student.
6. Prepare a SmartBoard or PowerPoint file with the discussion question at the end of the task.

Different Bases Task: Discovering Properties Student Sheet

Section1: The bottom row from each set of cards is an arithmetic sequence in which the relative difference is 1. In other words, it is the set of all integers (... - 3, -2, -1, 0, 1, 2, 3 ...). We will call this sequence the **adding world**. The top row of numbers is a geometric sequence with a common ratio of 3, 4, or 10. In other words, the top row could be powers of 3, 4, or 10. We will call this sequence the **multiplying world**. Fill in the chart below based on manipulating the cards given the mathematical operation. For example, if the bottom row is added, what happens to the top row? (You can make more cards using the blank cards if you want.)

When I am _____ in the Adding world (bottom row of numbers)....	..., I am _____ in the Multiplying world (top row of numbers)	Draw and Example
Adding	Multiplying	
Subtracting		
Multiplying (repetitive addition)		
Zero		
Multiplying by $\frac{1}{2}$		
Multiplying by -1		

Section 2

Make a table and graph for each function using the **adding world** as “x” and the **multiplying world** as “y”.

Base 3

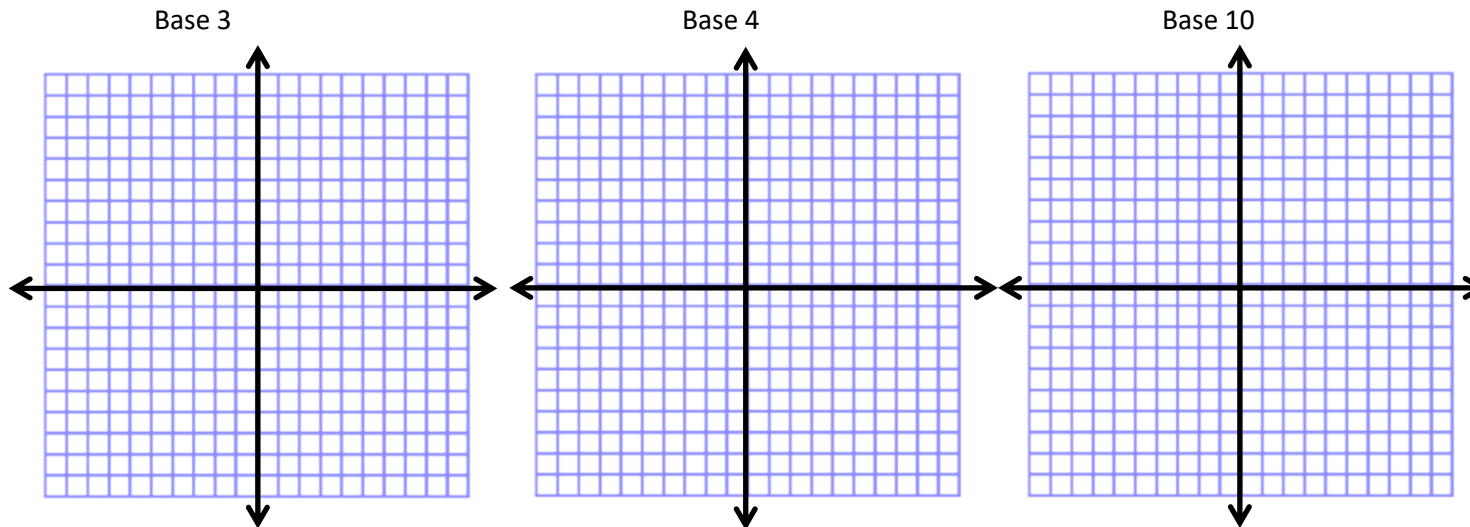
x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	$1/81$	$1/27$	$1/9$	$1/3$	1	3	9	27	81	243	728	2187

Base 4

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	$1/256$	$1/64$	$1/16$	$1/4$	1	4	16	64	256	1024	4096	16384

Base 10

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$	1×10^{-4}	1×10^{-3}	1×10^{-2}	1×10^{-1}	1	10	100	1000	10000	1×10^5	1×10^6	1×10^7



What is the domain and the range for each function?

What is the asymptote for each function?

What is the y-intercept for each function?

Section 3:

Make a table and graph for each function using the **multiplying world** as “x” and the **adding world** as “y”.

Base 3

x												
$f(x)$												

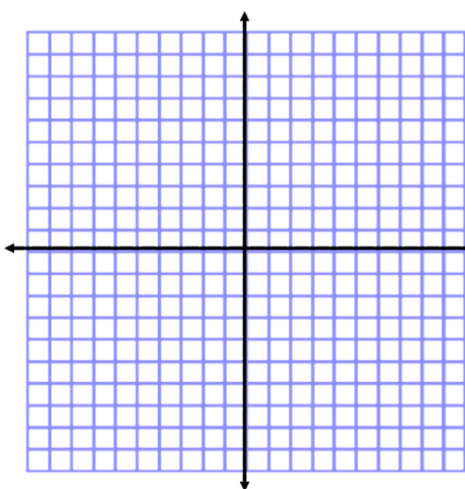
Base 4

x	1/256	1/64	1/16	¼	1	4	16	64	256	1024	4096	16384
$f(x)$	-4	-3	-2	-1	0	1	2	3	4	5	6	7

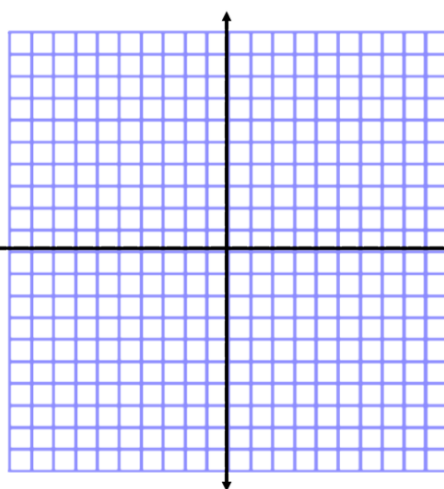
Base 10

x	1×10^{-4}	1×10^{-3}	1×10^{-2}	1×10^{-1}	1	10	100	1000	10000	1×10^5	1×10^6	1×10^7
$f(x)$	-4	-3	-2	-1	0	1	2	3	4	5	6	7

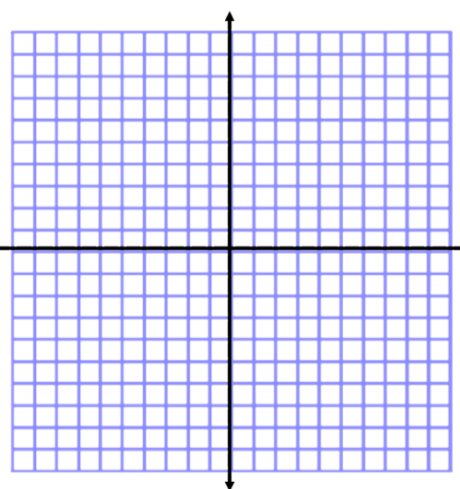
Base 3



Base 4



Base 10



What is the domain and the range for each function?

What is the asymptote for each function?

What is the x-intercept for each function?

How do the answers to these questions relate to the answers in section 2? Explain

Section 4:**Class discussion:**

Defining the logarithm: For any number in the geometric series [multiplying world], its logarithm is defined as the corresponding number in the arithmetic series [adding world]. (Smith & Confrey, 1994, p. 340)

Exponential functions: [***multiplying world***] = $b^{[\textit{adding world}]}$ or $m = b^a$

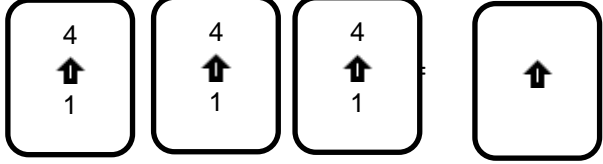
Logarithmic functions: [***adding world***] = $\log_b[\textit{multiplying world}]$ or $a = \log_b m$

Write the exponential and logarithmic equation for each set of cards.

Base	Exponential	Logarithmic
2	Example: $f(x) = 2^x$	Example: $g(x) = \log_2 x$
3		

4		
10		

Class Discussion Continued

When I am _____ in the Adding world (bottom row of numbers)....	..., I am _____ in the Multiplying world (top row of numbers)	Draw and Example
Add		
Subtract		
Multiply (repetitive addition)		
Zero		
Multiply by $\frac{1}{2}$		
Multiply by -1		

Class Discussion Continued

Characteristics of Exponential and Logarithmic Functions

Characteristic	Exponential of the form $f(x) = b^x$	Logarithmic of the form $g(x) = \log_b x$
Domain		
Range		
Intercepts		
Asymptote		

Different Bases Tasks Cards

Base 3

$\frac{1}{81}$ -4	$\frac{1}{27}$ -3	$\frac{1}{9}$ -2	$\frac{1}{3}$ -1	1 0	3 1
9 2	27 3	81 4	243 5	729 6	2187 7

Base 4

$\frac{1}{256}$ -4	$\frac{1}{64}$ -3	$\frac{1}{16}$ -2	$\frac{1}{4}$ -1	1 0	4 1
16 2	64 3	256 4	1024 5	4096 6	16384 7

Base 10

1×10^{-4} -4	.001 -3	$\frac{1}{100}$ -2	$\frac{1}{10}$ -1	1 0	10 1
100 2	1000 3	10000 4	1×10^5 5	1×10^6 6	1×10^7 7

Appendix F

pH and Logarithms: Base 10 with a Twist

Note: Students will have learned Arrhenius, hydronium, and hydroxide in chemistry, and they will be expected to use those terms in the mathematics classroom. In addition, the students will have approximated the pH values for several household items from the cabbage juice lab. Those values will be incorporated into this lesson.

Essential question: What happens to the pH value as the hydrogen ion concentration increases?

Learning Objective:

- Students will expand their knowledge of logarithms by applying them to the pH scale
- Students will examine multiple representation of the pH scale. (line graph, coordinate graph, and equation)
- Student will reinforce their vocabulary knowledge of chemistry terms.

Standards:

- SC6f. Obtain, evaluate, and communicate information about the properties that describe solutions and the nature of acids and bases.
- MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology. (Limit to exponential and logarithmic functions.)
- MGSE9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- MGSE9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (Limit to exponential and logarithmic functions.)

Materials:

- pH and logarithms task students sheet
- pH and logarithms task cards (cut out and ready to use)

Differentiated Instruction

- In the students-centered task, the teacher will monitor student progress and ask probing questions to help each group on-task and progressing through the activity

Assessment Strategies

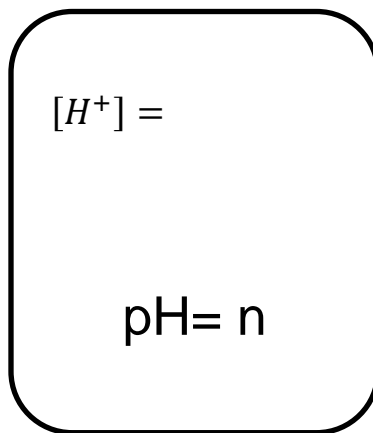
- Group interviews
- Monitoring activity

Lesson Preparation:

- Cut out pH and Logarithms Task Cards. Make enough sets of cards to allow groups of 2 or 3 to have a set.
- Copy pH and Logarithms Task Students sheet. Provide a copy for each student.
- Prepare a SmartBoard or PowerPoint file with the discussion question at the end of the task.

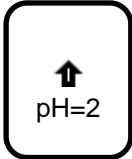
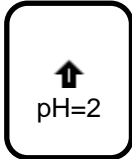
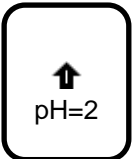
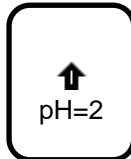
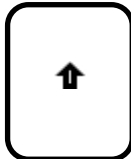
pH and Logarithms: Base 10 with a Twist Student Sheet

1. Find the cards that are completely filled out, and then determine a rule for the relationship between pH and hydrogen ion concentration $[H^+]$.



2. Create cards for pH from -4 to 15 and put the cards in order of the pH value.
 - a. As the pH increases, what is happening to the Hydrogen ion concentration $[H^+]$?
 - b. If one is added to the pH then the $[H^+]$ is _____ by _____.
(math operation) (number)
 - c. Which pH has a greater hydrogen ion concentration? How much bigger is it? (Use the rule established in part b)
pH = 3.6 or pH = 7.6
 - d. **Class Discussion:** What is a possible equation to map $[H^+]$ to pH? What is a possible equation to map pH to $[H^+]$?

Fill out the chart below using the cards created

When I am _____ the pH... (Adding world)	..., I am _____ the $[H^+]$ (Multiplying world)	Draw an Example
Add one	Divide by _____	 \div $+ 1 =$
Subtract one		
Add two		   $=$ 
Subtract 2		
zero		

If the one solution has a pH of 3 and another solution has a pH of 6, what is the relative difference in the hydrogen ion concentration?

3. Using the cards that you created, make a table, and plot the points on the coordinate plane. Let the x-value be $[H^+]$ and the y values be pH. In addition, let each black tick mark on the x-axis be one.

Describe the graph by finding the domain, range, intercepts, end-behavior, asymptote, and intervals of increase and decrease.

X	100	10	1	.1	.01	.001	.0001	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
$f(x)$	-2	-1	0	1	2	3	4	5	6	7	8	9

a. Domain $[H^+]$:

b. Range pH:

c. Intercept:

pH =

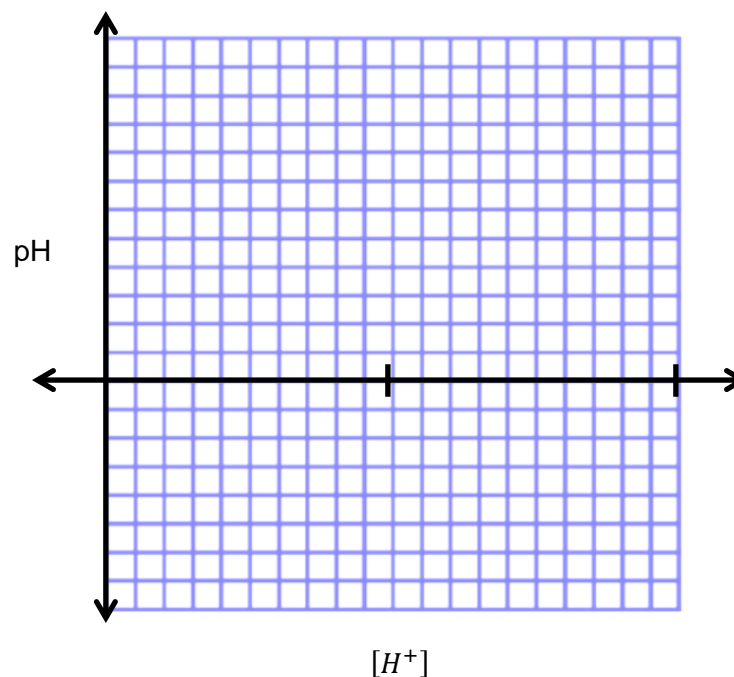
$[H^+] =$

d. Asymptote:

What does this mean? $[H^+]$ is greater than ____.

e. Interval of increase or decrease:

As pH increases, $[H^+]$ _____



4. Use the graph above to answer the following questions.

- a. If the pH is 0, then the hydrogen ion concentration would have to be ____.
- b. Can pH be negative? If so, the hydrogen ion concentration would have to be (greater or less) than one?
- c. What is the range for pH according to the graph?
- d. Can $[H^+]$ be negative? Explain.

5. Use the internet to answer the following questions

- a. What is the typical range for pH? From the Cabbage Juice Lab, did the pH values of those solutions lie within the typical range?
Can the pH value be outside of that range?
- b. What is the typical pH range for drinking water?
- c. Provide examples of solutions that can have a negative pH.
- d. Provide examples of solutions that can have a pH greater than 14.

$$[H^+] = 1 \times 10^{-6}$$

$$\text{pH}=6$$

$$[H^+] = 1 \times 10^{-5}$$

$$\text{pH} =$$

$$[H^+] = .0001$$

$$\text{pH}=4$$

$$[H^+] =$$

$$\text{pH}= 3$$

$$[H^+] = 1 \times 10^{-7}$$

$$\text{pH}=7$$

$$[H^+] = 1 \times 10^{-10}$$

$$\text{pH}=10$$

$$[H^+] =$$

$$\text{pH}=1$$

$$[H^+] = .01$$

$$\text{pH} =$$

$$[H^+] = 10$$

$$\text{pH} =$$

$$[H^+] = 1 \times 10^{-14}$$

$$\text{pH} = 14$$

$$[H^+] =$$

$$\text{pH} = 0$$

$$[H^+] =$$

$$\text{pH} = 4.5$$

$$[H^+] =$$

$$\text{pH} = -2$$

$$[H^+] =$$

$$\text{pH} = -3$$

$$[H^+] =$$

$$\text{pH} =$$

$$[H^+] =$$

$$\text{pH} =$$

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$$[H^+] =$$

$$\text{pH} =$$

Appendix G**What is a Logarithm? (Spotlight Task)**

As a society, we are accustomed to performing an action and then undoing or reversing that action. Identify the action that undoes each of those named.

Putting on a jacket

Opening a door

Walking forward

Depositing money in a bank

In mathematics we also find it useful to be able to undo certain actions.

What action undoes adding 5 to a number?

What action undoes multiplying a number by 4?

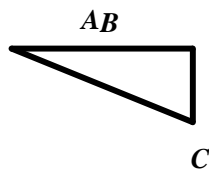
What action undoes squaring a number?

We say that addition and subtraction are inverse operations because one operation undoes the other. Multiplication and division are also inverse operations; squaring and taking the square root are inverse operations.

Inverse operations in mathematics help us solve equations. Consider the equation $2x + 3 = 35$. This equation implies some number (represented by x) has been multiplied by 2; then 3 has been added to the product for a result of 35. To determine the value of x , we subtract 3 from 35 to undo adding 3. This means that $2x$ must equal to 32. To undo multiplying the number by 2, we divide 32 by 2 and find the number represented by x is 16.

Explain how inverse operations are used in the solution of the following problems.

In right triangle ABC with right angle B, if BC is 8 cm and AC is 17 cm, determine the measure of angle A.



If $\sqrt{x + 8} = 10$, determine the value of x .

Solve $x^3 = 27$ for x .

Solve $2x = 10$ for x .

In problem 8 of Task 3, “Bacteria in the Swimming Pool,” we obtained the equation $1500(2)^t = 3000000$ to solve for t . This equation is equivalent to $2^t = 2000$. Why? While in Task 3 we had no algebraic way to solve this equation because we lacked a strategy to isolate the exponent t . Our goal in this current task is to continue our idea of “undoing” to solve an equation; specifically, we need to find an action that will undo raising 2 to a power. This action needs to report the exponent to which 2 has been raised in order to obtain 2000. In order to rewrite $2^t = 2000$ so t is isolated, we need to define logarithms. Logarithms allow us to rewrite an exponential equation so that the exponent is isolated. Specifically, if $a = b^c$, then “ c is the logarithm with base b of a ” and is written as $\log_b a = c$. (We read “ $\log_b a = c$ ” as “log base b of a is c .”)

Using logarithms we can write $2^t = 2000$ as $\log_2 2000 = t$. These two expressions are equivalent, and in the expression $\log_2 2000 = t$ we have t isolated. Although this is a good thing, we still need a way to evaluate the expression $\log_2 2000$. We know it equals the exponent to which 2 must be raised in order to obtain a value of 2000, but we still don’t know how to calculate this value. Hang on...we will get there in the next task! First some preliminary work must be done!

Let’s look at a few examples:

$10^2 = 100$ is equivalent to $\log_{10} 100 = 2$. Notice that 10 is the *base* in both the exponential form and the logarithmic form. Also notice that the logarithm is the exponent to which 10 is raised to obtain 100.

Evaluate $\log_4 64$. This question asks for the exponent to which 4 is raised to obtain 64. In other words, 4 to what power equals 64? _____

Consider the following problem: $\log_2 l = 4$. This equation is equivalent to $2^4 = l$; thus $l = 16$.

The relationship between exponents and logarithms must be understood clearly. The following practice problems will help you gain this understanding.

Rewrite each exponential equation as a logarithmic equation.

$$6^2 = 36$$

$$10^3 = 1000$$

$$25^{\frac{1}{2}} = 5$$

Rewrite each logarithmic equation as an exponential equation.

$$\log_4 16 = 2$$

$$\log_6 1 = 0$$

$$\log_3 l = t$$

Evaluate each of the following.

$$\log_{10}(0.1) \quad \log_3 81 \quad \log_{216} \text{---}^1 \quad \log_5 5$$

Between what two whole numbers is the value of $\log_3 18$?

Between what two whole numbers is the value of $\log_2 50$?

Solve each logarithmic equation for x . a)

$$\log_9 81 = x$$

$$\text{b) } \log_2 32 = x$$

$$\text{c) } \log_7 1 = x$$

$$\text{d) } \log_8 x = 3$$

$$\text{e) } \log_5(3x + 1) = 2$$

$$\text{f) } \log_6(4x - 7) = 0$$

Hopefully you now have an understanding of the relationship between exponents and logarithms.

In logarithms, just as with exponential expressions, any positive number can be a base except 1 (we will explore this fact later). Logarithms which use 10 for the base are called common logarithms and are expressed simply as $\log x$. It is not necessary to write the base. Calculators are programmed to evaluate common logarithms.

Use your calculator to evaluate $\log 78$. First think about what this expression means.

Understanding logarithms can help solve more complex exponential equations. Consider solving the following equation for x :

$$10^x = 350$$

We know that $10^2 = 100$ and $10^3 = 1000$ so x should be between 2 and 3. Rewriting $10^x = 350$ as the logarithmic equation $x = \log 350$, we can use the calculator to determine the value of x to the nearest hundredth.

Solve each of the following for x using logarithms. Determine the value of x to the nearest hundredth.

$$1. 10^x = 15$$

$$2. 10^x = 0.3458$$

$$3. 3(10^x) = 2345$$

$$4. 2(10^x) = -6538$$

Logarithms that use the irrational number e as a base are of particular importance in many applications. Recall an irrational number is represented by a non-terminating, non-repeating decimal number. The value of e is 2.718281828.... The function $y = \log_e x$ is the natural logarithmic function and has a base of e . The shorthand for $y = \log_e x$ is $y = \ln x$. Calculators are also programmed to evaluate natural logarithms.

Consider $\ln 34$ which means the exponent to which the base e must be raised to obtain 34. The calculator evaluates $\ln 34$ as approximately 3.526. This value makes sense because $e^{3.526}$ is approximately 33.9877, a value very close to 34!

Evaluate $\ln 126$. Use an exponential expression to confirm your solution makes sense.

Evaluate $\ln e$. Explain why your answer makes sense.

If $\ln x = 7$, determine the value of x to the nearest hundredth. HINT: Write the logarithmic equation in exponential form.

If $e^x = 85$, determine the value of x to the nearest hundredth. HINT: Write the exponential equation in logarithmic form.

The cards you have been given are to be sorted. There will be six matches of five cards each. You will see a verbal description of the exponential function, a verbal description of the logarithmic function that means the same thing, the logarithmic equation written out, the exponential equation written out, and the solution to the equations. Make the matches, and then be prepared to tell:

- a) Of the two equations that you saw, the exponential and the logarithmic, which one helped you find the solution the easiest?
- b) How does the solution that you found work for both the logarithmic and the exponential equation?
- c) Which ones could you have solved without any work at all except just using your calculator?

STUDENT COPY-CARD SORT-SHOULD BE CUT APART BEFORE BEING PLACED INTO
STUDENT HANDS

<p>A</p> <p>The logarithm, base two, of 16 is some number.</p>	<p>f</p> <p>Two to the power of some number is 16.</p>	<p>U</p> <p>$\log_2 16 = x$</p>	<p>Q</p> <p>4</p>	<p>S</p> <p>$2^x = 16$</p>
<p>P</p> <p>The logarithm, base 16, of 2 is a number.</p>	<p>B</p> <p>Sixteen to the power of some number is 2.</p>	<p>O</p> <p>$\log_{16} 2 = x$</p>	<p>W</p> <p>$\frac{1}{4}$</p>	<p>G</p> <p>$16^x = 2$</p>
<p>Ω</p> <p>The logarithm with what base of 2 is 16?</p>	<p>M</p> <p>Two to the power of some number is 16.</p>	<p>N</p> <p>$\log_x 2 = 16$</p>	<p>E</p> <p>$\pm 2^{\frac{1}{16}}$</p>	<p>V</p> <p>$x^{16} = 2$</p>
<p>L</p> <p>The logarithm with what base of 16 is 2?</p>	<p>F</p> <p>Some number to the power of 2 is 16.</p>	<p>T</p> <p>$\log_x 16 = 2$</p>	<p>I</p> <p>± 4</p>	<p>Z</p> <p>$x^2 = 16$</p>
<p>μ</p> <p>The logarithm, base two, of some number is 16.</p>	<p>C</p> <p>Two to the power of 16 is some number.</p>	<p>R</p> <p>$\log_2 x = 16$</p>	<p>K</p> <p>65,536</p>	<p>J</p> <p>$2^{16} = x$</p>
<p>H</p> <p>The logarithm, base 16, of some number is 2.</p>	<p>X</p> <p>Sixteen to the power of 2 is some number.</p>	<p>Y</p> <p>$\log_{16} x = 2.$</p>	<p>θ</p> <p>256</p>	<p>Я</p> <p>$16^2 = x$</p>

Appendix H

IRB Forms

Kennesaw State University IRB approval

8/8/2017

Andrew Smith

RE: Your follow-up submission of 8/8/2017, Study #18-030: Using STEM to Prepare Students for Success

Dear Mr. Smith,

Your application for the new study listed above has been administratively reviewed. This study qualifies as exempt from continuing review under DHHS (OHRP) Title 45 CFR Part 46.101(b)(1) - normal educational practices. The consent procedures described in your application are in effect. You are free to conduct your study.

NOTE: All surveys, recruitment flyers/emails, and consent forms must include the IRB study number noted above, prominently displayed on the first page of all materials.

Please note that all proposed revisions to an exempt study require IRB review prior to implementation to ensure that the study continues to fall within an exempted category of research. A copy of revised documents with a description of planned changes should be submitted to irb@kennesaw.edu for review and approval by the IRB.

Thank you for keeping the board informed of your activities. Contact the IRB at irb@kennesaw.edu or at (470) 578-2268 if you have any questions or require further information.

Sincerely,

Christine Ziegler, Ph.D.
KSU Institutional Review Board Chair and Director

Floyd County Schools IRB Approval

Floyd County Schools Administrative Rule for Research Procedures

D. Applicant Agreement

FLOYD COUNTY SCHOOLS
Access to Confidential Data
Applicant Agreement

Research Applicant Andrew Smith
 Research Title Using STEM to Prepare Students for Success
 Home Address 201 Dodd Street
 City/State/Zip Rome, Ga 30165
 Employer Floyd County Schools
 Telephone: Work 706-236-1844 Home 706-766-2402
 Fax 706-236-1846 E-mail ansmith@floydboe.net

I understand that any unauthorized disclosure of confidential information is illegal as provided in the Family Educational Rights and Privacy Act of 1973 (FERPA) and in the implementing federal regulations found in 34 CFR Part 99. I understand that participation in a research study by students, parents, and school staff is strictly voluntary. I further understand and agree that Floyd County Schools may, in its sole discretion, terminate any research or project and may revoke its consent and permission for Research Applicant to continue research within the School District.

In addition, I understand that any data, datasets or outputs that I, or any authorized representative, may generate from data collection efforts throughout the duration of the research study are confidential and the data are to be protected. I will not distribute to any unauthorized person any data or reports that I have access to or may generate using confidential data. I also understand that students, schools, or the district may not be identified in the research report. Data with names or other identifiers (such as student numbers) will be made indecipherable and disposed of when their use is complete.

I understand that acceptance of this request for approval of a research project in no way obligates the Floyd County Schools to participate in the research. I also understand that approval does not constitute commitment of resources or endorsement of the study or its findings by the school system or by the Board of Education. I further agree to immediately terminate said research project immediately if the School District revokes its permission for me to conduct the research study.

If the research project is approved, I agree to abide by standards of professional conduct while working in the schools. I understand that failure to do so could result in termination of the research study.

I agree to send a complete copy of the study results to the Department of Academics, Director for School Improvement, after completion of the study for any future use to the Floyd County Schools. I understand that the study is not complete until this report has been provided to Floyd County Schools.

<u>Andrew Smith</u> Research Applicant Signature	<u>7/11/2017</u> Date
<u>J. Alcorn</u> Signature of Your Administrator Sponsor (i.e., Your Principal)	<u>7/26/17</u> Date
<u>Don Quame</u> Signature of Sponsoring Agency (i.e., College Professor)	<u>7/11/2017</u> Date
<u>Barbara Smith</u> Signature of Floyd County Schools Department of Academics Granting Approval of the Attached Research Proposal Once Signed, Applicant May Proceed with Research	<u>7/31/17</u> Date

Appendix I

Pacing Guide for Traditional Unit

Day	Algebra II	Chemistry
1	Lesson 1: Investigating Exponential Growth and Decay (2 Days)	
2		
3	Lesson 2: Graphs of Exponential Functions (2 Days)	Lesson 1: Discovery Activity of pH using Red Cabbage Juice, Beet Juice and Blueberry juice which are natural pH indicators (2 days)
4		
5	Lesson 3: Bacteria in a Swimming Pool (1 day)	Lesson 2: Develop understanding of chemistry language for Acid/Bases (1 day)
6	Lesson 4: What is a Logarithm? (1 days	Lesson 3: Direct Instruction of characteristics of acids, bases, and salts with emphasis on Arrhenius acid and bases. (1 day)
7	Lesson 5: Evaluating, Estimating, and comparing (pH not included) (4 days)	Lesson 4: Simulation of Molecular interactions of acids and bases (includes methods of analysis to determine various properties of the acids and bases) (2 days)
8		
9		Lesson 5: Hydronium ion concentration and pH calculation Problem Solving (2 days)
10		
11	Lesson 6: Solving simple exponential and logarithmic equations (pH not included) (one day)	Lesson 6: Identify an acid and base produced and distributed commercially and identify practical characteristics that make them desirable. (3-4 Days)
12	Lesson 7: Graphing basic logarithmic and exponential functions (2 days)	
13		
14	Lesson 8: Review and Chemistry Review (Stations) (1 Day)	Lesson 7: Review and Chemistry Review—Nearpod Skill Builder Activity
15	Logarithms Assessment	pH assessment

Pacing Guide for integrated STEM Unit

Day	Algebra II	Chemistry
1	Lesson 1: Base 2 Card Task (Two Days)	
2		
3	Lesson 2: Dealing with Multiple Bases task (Two Days)	Lesson 1: Discovery Activity of pH using Red Cabbage Juice, Beet Juice and Blueberry juice which are natural pH indicators (2 days)
4		
5	Lesson 3: pH and logarithms (base 10 with a twist) task (1 day)	Lesson 2: Develop understanding of chemistry language for Acid/Bases (1 day)
6	Lesson 4: Evaluating logarithms with rational answers and converting forms (1day)	Lesson 3: Direct Instruction of characteristics of acids, bases, and salts with emphasis on Arrhenius acid and bases. (1 day)
7	Lesson 5: Evaluating, Estimating, and comparing (pH included) (4 days)	Lesson 4: Simulation of Molecular interactions of acids and bases (includes methods of analysis to determine various properties of the acids and bases) (2 days)
8		Lesson 5: Hydronium ion concentration and pH calculation Problem Solving (2 days)
9		
10		
11	Lesson 6: Solving simple exponential and logarithmic equations (pH included) (one day)	Lesson 6: Identify an acid and base produced and distributed commercially and identify practical characteristics that make them desirable. (3-4 Days)
12	Lesson 7: Graphing basic logarithmic and exponential functions (2 days)	
13		
14	Lesson 8: Review and Chemistry Review (Stations) (1 Day)	Lesson 7: Review and Chemistry Review—Nearpod Skill Builder Activity
15	Logarithms Assessment	pH assessment

Appendix J**Conceptual Understanding of Logarithms (Weber, 2002) Codebook**

Domain	Level of Conceptual Understanding	Evidence
Logarithms	Exponentiation as an action	<ul style="list-style-type: none"> A student evaluates exponential functions with only positive integer exponents. A student understands exponential expressions such as 2^3 as $2*2*2$ A student has not shown that they can reverse the process of exponentiation (evaluate logarithms).
	Exponentiation as a process	<ul style="list-style-type: none"> A student reasons verbally or through written explanation that an exponential function is increasing or decreasing based on repeated multiplications of the base. A student can reverse the process of exponentiation verbally or through written explanation by evaluating logarithms in which the exponent is a natural number (Example: $\log_2 8$)
	Exponential expressions as the result of a process	<ul style="list-style-type: none"> A student can explain verbally or through written explanation that the relationship between exponent and logarithmic properties and apply common properties of logarithms such as the product property And a student's knowledge of logarithms is restricted to natural numbers, or it has extended to negative, rational, or irrational numbers by using given rules (Example: A student states that $\log_2 \frac{1}{8} = -3$ because $2^{-3} = \frac{1}{2^{-3}} = \frac{1}{8}$)
	Generalization	<ul style="list-style-type: none"> A student's knowledge of logarithms is not restricted to natural numbers or given rules. It has extended to negative, rational, or irrational numbers, and the student can articulate the meaning of logarithms containing negative, rational, or irrational exponents (Example: A student states that $\log_4 2 = \frac{1}{2}$ because half a factor of 4 is 2)

Note: From "Students' Understanding of Exponential and Logarithmic Functions" by K. Weber (2002)

Conceptual Understanding of pH (Park & Choi, 2012) codebook

Type	Evidence
Object Type 1	<ul style="list-style-type: none"> • pH is a measure of acidity • pH is a qualitative descriptor • Differences in pH values is described qualitatively such as dangerous or safe
Operation Type 2	<ul style="list-style-type: none"> • pH is a measure of acidity • pH values are proportional to a base (pH of 1 and pH of 3 have a relative difference of 100) • Base of 10 is memorized
Operation Type 3	<ul style="list-style-type: none"> • pH is a measure of acidity • can manipulate the logarithmic formula • Obtain proportional differences • Mathematical understanding separate from scientific meaning.
Operation Type 4	<ul style="list-style-type: none"> • PH is a measure of Hydrogen ion concentration • pH values are proportional to a base (pH of 1 and pH of 3 have a relative difference of 100) • Base of 10 is memorized
Operation Type 5	<ul style="list-style-type: none"> • pH is a measure of Hydrogen ion concentration • can manipulate the logarithmic formula • Obtain proportional differences • Mathematical understanding connected to scientific meaning.

Note: From “Analysis of students understanding of science concept including mathematical representations: pH values and the relative difference of pH values.” By E. Park & K. Choi, 2012 International Journal of Science and Mathematics Education, and “STEM integration in K-12 education: Status, prospects, and an agenda for research” by Honey, M., Pearson, G., & Schweingruber, H. (Eds.). (2014) National Academies Press.

Transfer of Knowledge and Different Representations Codebook

Type	Evidence
Conceptual understanding expressed through connecting different representations (Panasuk, 2010)	<ul style="list-style-type: none"> A student makes connections between two or more representations (Written, numerical, algebraic, and graphical). For example, a student may conclude that the function $f(x) = \log_2 x$ has a x-intercept of one because $\log_2 1 = 0$.
Transfer of knowledge between content domains (Honey, Pearson, & Scheweingruber, 2014).	<ul style="list-style-type: none"> A student uses his or her understanding of logarithms to explain pH or his or her understanding of pH to explain logarithms. For example, a student might use the processes learn in mathematics to evaluate pH, or a student might understand pH to be a base 10 system and conclude that the common logarithm should be used in the formula to calculate pH. A student connects his or her understanding of pH to logarithms. For example, a student might understand the range of pH in chemistry to typically be 0 to 14 and understand that the mathematical formula for pH allows pH to range from negative infinity to infinity. Through this insight, the student gains a deeper understanding of the functional relationship between pH and hydrogen ion concentration and the meaning of concentration.

Appendix K

Logarithms and pH Assessment Rubrics

Levels of Conceptual Understanding

Logarithms (Weber, 2002)

- Exponentiation as an action: A student has the ability to evaluate exponents by repeated multiplication. The action of the students is specific to positive integer exponents and is limited to computing and manipulating formulas that involve integers.
- Exponentiation as a process: A student has the ability to internalize the process of evaluating integer exponents and can reverse that process (logarithms). These students can generalize their thinking to a functional representation such as 2^x and understand some of the function's properties. For example, students can understand that 2^x is an increasing function because it involves repeatedly multiplying by two.
- Exponential expressions as the result of a process: At this level of understanding, students start to view a correspondence between a power and its product. For example, students internalize that 2^3 is 8 without having to multiply 2 to itself three times. Students at this level of understanding are able to reason and understand properties of exponents such as $b^x b^y = b^{x+y}$.
- Generalization: At this level of understanding students are able to expand their knowledge beyond natural numbers. Students are able to compute values that involve negative and rational exponents and understand that powers can equal rational and irrational values.

pH (Park & Choi, 2014)

- Type 1 Object: Students understand pH as a characteristic or measure of acidity.
- Type 2 Operation: Students understand pH as a characteristic but can perform operations with pH. These students measure pH with proportional differences. A pH of 3 and a pH of 5 have a difference of 2.
- Type 3 Operation: Students understand pH as a qualitative value but can perform operations with pH. These students measure pH with proportional differences in logarithms. A pH of 3 has 100 times more concentration of hydrogen ions than a pH of 5.
- Type 4 Function: pH is a measure of hydrogen ion concentration and perform calculations like students in Operation 1.
- Type 5 Function: pH is a measure of hydrogen ion concentration and perform calculations like students in Operation 2.

pH and logarithmic levels of understanding

- Undeveloped (1): Student indicates an understanding of pH at the object level or logarithms at the "exponentiation as an action" level of understanding
- Applied (2): Student indicates an understanding pH at the operation 1 level or logarithms at the "exponentiation as a process" level of understanding. The student treats a logarithm as a tool to solve and not as a function.

- Developed (3): The student indicates an understanding of pH at the operation 2 or functional type 1 level or logarithms at the “exponentiation as the result of a process” level of understanding
- Advanced (4): The student indicates an understanding of pH at the functional type 2 level or logarithms at the “generalization” level of understanding

Level	Logarithms		pH
Below Basic			
Undeveloped (Basic)	Students can evaluate powers with integer exponents		pH is seen solely as a characteristic of a substance and not a measure of hydrogen ion concentration.
Applied	Students can evaluate and reverse the process (logarithms) of powers with integer exponents. In addition, students can reason about certain properties such as increasing or decreasing		pH is still seen as a characteristic, but students use proportional reasoning (not logarithmic). Example, pH of 3 and pH of 1 have a relative difference of 3 (3/1)
Developed	Students understand powers as a certain number of factors of the base. However, their domain is still restricted to the natural numbers		<p>pH is still seen as a characteristic, but students use proportional differences in logarithms. Example, pH of 3 and pH of 1 have a relative difference of 100 (1000/10).</p> <p>Or</p> <p>pH is seen as a measure of concentration, and students use proportional reasoning (not logarithmic). Example, pH of 3 and pH of 1 have a relative difference of 3 (3/1)</p>
Advanced	Students understand powers that are negative, fractions, or irrational		pH is seen as a measure of concentration, and students use proportional differences in logarithms. Example, pH of 3 and pH of 1 have a relative difference of 100 (1000/10).

Terms:

$$\log_b x = y$$

- b is the base
- x can be referred to as the argument, geometric value, or value in the multiplying world
- y can be referred to as the exponent, arithmetic value, or value in the adding world

Validation and Reliability:

Each rubric was made based on the levels of understanding of logarithms and pH created by Weber (2002) and Park and Choi (2012). After the pilot study, 15 randomly selected logarithmic in a mathematics context tests were given to three mathematics certified teachers

for grading, and 15 randomly selected logarithms in a science context tests were given to two mathematics and one science certified teachers for grading. Once the tests were graded individually, the groups discussed any differences until a consensus was reached. Then the process was repeated for 15 more randomly selected tests.

NOTE: If a student's answer shows understanding from multiples levels, the highest level was chosen.

MW1 Rubric

Define or explain a logarithm.

Level 4: Students showing an advanced level of understanding of logarithms will be able to define a logarithm in one of two ways. First, students taught the traditional correspondence approach will define logarithms as the power the base must be raised to obtain a given value (Confrey & Smith, 1995). Secondly, students taught a covariational approach will define a logarithm as the corresponding value in an arithmetic sequence for a given value in the geometric sequence (Smith & Confrey, 1994). Each of these definitions are considered developed, because the students understand that the relationship between the argument and the exponent is defined by the base of the logarithm.

Level 3: Students showing a developed understanding of logarithms will have an acceptable definition of a logarithm. A logarithm can be defined as an exponent or as the inverse of an exponential equation. These definitions are not considered developed, because the student fails to mention the relationship between the base, argument, and exponent. However, these definitions are considered acceptable, because they are included in the Georgia Standards of Excellence (2015).

Level 2: Students showing an applied but undeveloped understanding of logarithms will associate the logarithm with an application. The application can be in the context of chemistry or a property in mathematics. In other words, the student uses a logarithm as a tool to solve and does not understand that a logarithm is a function.

Level 1: Students showing an undeveloped understanding of logarithms will confuse the argument, base, and exponent. However, they do express some understanding, because they show that the logarithm is a relationship between quantities (argument, base, and exponent).

MW1

Score	Description
0	No response, unrelated definition, or extremely vague
1	<p>Undeveloped: Student makes an incorrect, but related explanation.</p> <ul style="list-style-type: none"> • A student confuses the argument and the exponent • A student uses incorrect operations between the base and the argument • A student is unclear about whether the base or the argument is being raised to a certain power • A student mentions that a logarithm is equal to something other than an exponent • A student uses improper terms such as reciprocal instead of inverse • A student uses unclear language in which the definition of a logarithm cannot be translated • A student mentions that a logarithm is a function, equation, or relation, but no further description is provided that develops the meaning of a logarithm (see level 3 and 4 for definitions).
2	<p>Applied: Student uses a logarithm as a tool to solve.</p> <ul style="list-style-type: none"> • A student mentions the application of a logarithm such as pH, solving equations, or finding exponents but no further description is provided that develops the meaning of a logarithm (see level 3 and 4 for definitions). • A student mentions a method for solving logarithms, but no further description is provided that develops the meaning of a logarithm (see level 3 and 4 for definitions). • A student mentions that a logarithm is another way to write a problem or exponential function.
3	<p>Developed: A student understands that a logarithm is an exponent or the inverse of an exponential function, but he or she does not state that a logarithm is a function between two sets.</p> <ul style="list-style-type: none"> • A student states that a logarithm is an exponent • A student states that a logarithm is the inverse of an exponential function • A student states their answer in the form of a question. What power must the base be raised to get the argument?
4	<p>Advanced: A student understands that a logarithm is a function between two set (arithmetic and geometric sequence or exponents and powers)</p> <ul style="list-style-type: none"> • A student states that a logarithm is the corresponding value in an arithmetic sequence (adding world) for a given value in a geometric sequence (multiplying world) (Smith & Confrey, 1994) • A student states that a logarithm is the corresponding exponent or power that a base must be raised for a given value. <p>Note: Students can refer to an exponent as the number of times a base is multiplied to itself.</p>

MW2

Explain the meaning of the following expression using complete sentences.

$$\log_2 5$$

Level 4: Students showing an advanced level of understanding of logarithms will be able to define $\log_2 5$ in one of two ways. First, students taught the traditional correspondence approach will define $\log_2 5$ as the power that two must be raised to obtain 5. Secondly, students taught a covariational approach will define a logarithm as the corresponding value in an arithmetic sequence for five in the geometric sequence with a common ratio of 2. Each of these definitions are considered developed, because the students understand that the relationship between five and the exponent is defined by the base of two.

Level 3: Students showing a developed understanding of logarithms will have an acceptable explanation of $\log_2 5$. These students will explain this logarithm as the exponent of two or as the inverse of two raised to a power. These explanations are not considered developed, because the student fails to mention the relationship between the base, the argument, and the exponent.

Level 2: Students showing an applied but undeveloped understanding of $\log_2 5$ only try to evaluate the logarithm. In addition, students that use unclear language have not communicated through writing that they can properly communicate the meaning of $\log_2 5$.

Level 1: Students showing an undeveloped understanding of $\log_2 5$ will confuse the argument, base, and exponent. However, they do express some understanding, because they show that the logarithm is a relationship between quantities (argument, base, and exponent). In addition, these students may restate the problem without explaining the meaning.

MW2

Score	Description
0	No response, unrelated definition, or extremely vague
1	Undeveloped: A student makes an incorrect, but related explanation. <ul style="list-style-type: none"> • A student confuses the exponent and the argument • A student confuses that base with the exponent or the argument • A student uses improper terms for the base, argument, or exponent • A student uses improper operations • A student restates the problem as log base 2 of 5
2	Applied: Student uses a logarithm as a tool to solve. <ul style="list-style-type: none"> • A student describes how to solve the logarithm • A student estimates the value of the logarithm between 2 and 3. • A student mentions that the logarithm has a base of 2 and you are trying to get 5, but the student does not mention exponents
3	Developed: A student understands that a logarithm is an exponent or the inverse of an exponential function, but he or she does not state that a logarithm is a function between two sets. <ul style="list-style-type: none"> • A student states that the logarithm is the power that 2 must be raised, but does not mention the argument given (5) • A student understands the base is 2 and the argument is 5 but does not mention the operation to get from 2 to 5. • A student restates the problem as a question. (Two to what power is 5?)
4	Advanced: A student understands that a logarithm is a function between two sets (arithmetic and geometric sequence or exponents and powers) <ul style="list-style-type: none"> • A student states that the logarithm is the power that 2 must be raised to obtain 5 • A student states that the logarithm is the corresponding value in the adding world for 5 in the multiplying world (base 2 system)

MW3

Is it possible to evaluate $\log_2(-5)$? Why or Why not?

Level 4: Students showing an advanced level of understanding of logarithms will be able to explain that $\log_2(-5)$ cannot be evaluated. First, students taught the traditional correspondence approach will explain that a positive base to any power will not be negative. Second, students taught a covariational approach will explain that the argument (multiplying world) cannot be negative, because dividing by increasing powers of 2 will only approach zero. Each of these explanations are considered developed, because the students understand that the argument must be positive if the base is positive. A positive base raised to any power will result in a positive value.

Level 3: Students showing a developed understanding of logarithms will have an acceptable explanation of $\log_2(-5)$. These students will communicate that the argument cannot be negative but will fail to elaborate or explain this logarithm as the exponent of two or as the inverse of two raised to a power. These explanations are not considered developed, because the student fails to mention the relationship between the base, the argument, and the exponent.

Level 2: Students showing an applied but undeveloped understanding of $\log_2(-5)$ only try to evaluate the logarithm. They are focused on the procedure and not the meaning of the mathematical expression. In addition, students that use unclear pronouns have not communicated through writing that they can properly communicate the meaning of $\log_2(-5)$.

Level 1: Students showing an undeveloped understanding of $\log_2(-5)$ will confuse the argument, base, and exponent. However, they do express some understanding, because they show that if negative five was an exponent of two then one thirty-second is the result.

MW3

Score	Description
0	<p>No response, unrelated definition, or extremely vague</p> <ul style="list-style-type: none"> • Yes or No with no explanation • Yes, with an unrelated explanation • No with an unrelated explanation
1	<p>Undeveloped: Student makes an incorrect, but related explanation.</p> <ul style="list-style-type: none"> • Yes, with a related explanation <ul style="list-style-type: none"> ○ A student states using a number line to evaluate the logarithms ○ A student evaluates the logarithm by using negative five as an exponent or determines that the answer will be a fraction or decimal • No with an incorrect but related explanation
2	<p>Applied: Student uses a logarithm as a tool to solve.</p> <ul style="list-style-type: none"> • No with vague explanation, but could be considered correct <ul style="list-style-type: none"> ○ A student states a negative cannot be evaluated ○ A student states it cannot be negative ○ A student states it can't go past zero ○ A student states that the argument cannot be negative
3	<p>Developed: A student understands that a logarithm is an exponent or the inverse of an exponential function, but he or she does not state that a logarithm is a function between two sets.</p> <ul style="list-style-type: none"> • A student answers with no and states that multiplying positives does not result in a negative • A student states that exponents can only create fractions and not negatives
4	<p>Advanced: Students understands that a logarithm is a function between two set (arithmetic and geometric sequence or exponents and powers)</p> <ul style="list-style-type: none"> • No with developed explanation <ul style="list-style-type: none"> ○ A student's reasoning is a positive base to any power cannot be negative ○ A student states that the argument (multiplying world) cannot be negative, because multiplying or dividing by powers of 2 will not be negative

MW4

Explain the range of $f(x) = \log_3 x$ by filling in the blanks and completing the sentence.

The range of the function is _____ (minimum) to _____ (maximum),
because ...

Level 4: Students showing an advanced level of understanding of logarithms will be able to identify and explain the range of a logarithmic function. First, students taught the traditional correspondence approach will explain that the range is from negative infinity to infinity or all real numbers, because an exponent can be any real number. Secondly, students taught a covariational approach will explain that the range is from negative infinity to infinity or all real numbers, because the arithmetic sequence (adding world) can be any real number. Each of these definitions are considered developed, because the students understand that a positive base can be raised to any real number or that the rate of change in the arithmetic sequence can result in any possible number.

Level 3: Students showing a developed understanding of logarithms will understand that the range is from negative infinity to infinity, but they will not explicitly mention exponents or the adding world. They could mention that the graph of a logarithm will go up and down forever. In addition, a student can respond with zero to infinity and still be considered to have a developed understanding. Even though zero to infinity is not the range of a logarithmic function, a student can provide a developed explanation for their answer such as a power of a positive base will always be greater than zero or the geometric sequence multiplying world will never be negative. (Remember the objective of the test is to measure a student's conceptual understanding of logarithms and not their definition of domain and range.)

Level 2: Students showing an applied but undeveloped understanding of logarithms will answer with negative infinity to infinity or zero to infinity but provide vague explanation or an unclear pronoun.

Level 1: Students showing an undeveloped understanding of logarithms will provide a related answer with no explanation or an incorrect explanation.

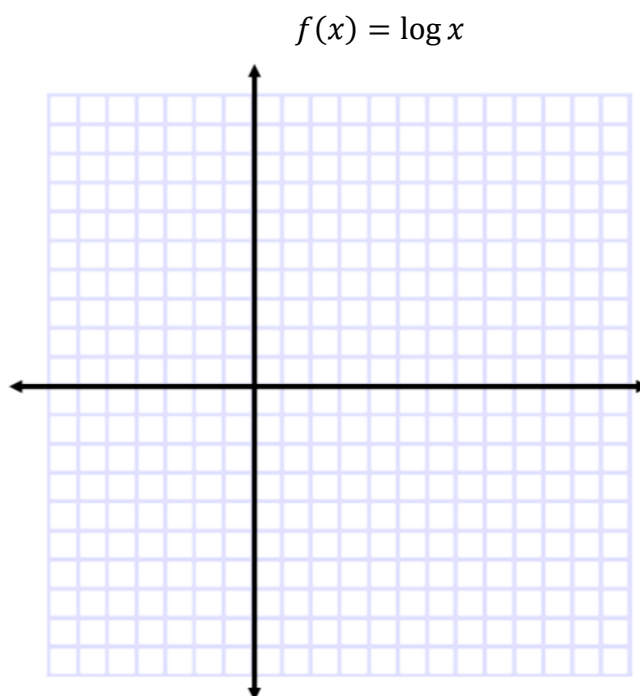
MW4

Score	Description
0	No response or unrelated explanation
1	Undeveloped: A student makes an incorrect, but related explanation. <ul style="list-style-type: none"> • $(0, \infty)$ or $(-\infty, \infty)$ and no explanation or incorrect explanation
2	Applied: A student uses a logarithm as a tool to solve. <ul style="list-style-type: none"> • $(-\infty, \infty)$ with vague explanation, but could be considered correct <ul style="list-style-type: none"> ○ A student states that it can be any real number ○ A student states it goes in both directions • $(0, \infty)$ with vague explanation, but could be a proper justification for their answer <ul style="list-style-type: none"> ○ A student states that it gets closer to zero ○ A student refers to an exponential graph or number line
3	Developed: A student understands that a logarithm is an exponent or the inverse of an exponential function, but he or she does not state that a logarithm is a function between two sets. <ul style="list-style-type: none"> • $(-\infty, \infty)$ and an undeveloped correct explanation <ul style="list-style-type: none"> ○ A student does not clearly mention the adding world or exponents, but provide a clear explanation by referring to a number line or graph ○ A student states that an exponent can be any number. • $(0, \infty)$ and developed explanation <ul style="list-style-type: none"> ○ A student specifies the multiplying world or that a number to a power can never be lower than zero. ○ A student shows an understanding of a relationship between two sets.
4	Advanced: A student understands that a logarithm is a function between two sets (arithmetic and geometric sequence or exponents and powers) <ul style="list-style-type: none"> • $(-\infty, \infty)$ and developed correct explanation <ul style="list-style-type: none"> ○ A student states that the output of a logarithms is an exponent and the base can be raised to any power.

MG2:

17. Using the seven different values listed below for x , sketch a graph of the following function.

x	.001	0.1	10	1000	10023	1000000	10^8
y							



Level 4: A student that demonstrates an advanced level of understanding will be able to graph the function, label the axes with a proper scale, and graph a non-linear function (Park & Choi, 2012). In addition, the students will be able to estimate the logarithms at non-integer powers (Weber, 2002). Therefore, a student that provides a well labeled and correct graph with a correctly fill table shows and advanced level of understanding.

Level 3: A student that demonstrates a developed level of understanding will be able to properly fill in the table for most values and be able to provide an appropriate graph with no scale.

Level 2: A student that demonstrates an applied but undeveloped understanding will be able to provide a related graph or fill in approximately half of the values in the table.

Level 1: According to Park and Choi (2012), a student that demonstrates a memorized response or a descriptive response where the graph shows at least an increasing or decreasing relationship shows a limited understanding of logarithms. Therefore, a student that draws at least a related graph and cannot fill in the table is considered a level one. In addition, a student that miscalculates most of the values in the table indicates an “Exponentiation as an Action”

level (Weber, 2002). In other words, a student is only able to evaluate the logarithm for values that result in whole numbers.

MG2

Score	Description
0	No response or unrelated response
1	Undeveloped <ul style="list-style-type: none"> • A logarithmic graph or linear graph with a logarithmic scale and... <ul style="list-style-type: none"> ○ More than 5 mistakes in the table ○ The shape of the graph is approximately correct • Exponential Graph or pH graph ($pH = -\log[H^+]$) <ul style="list-style-type: none"> ○ More than 3 mistakes in the table ○ Graph is approximately correct
2	Applied <ul style="list-style-type: none"> • A logarithmic graph or linear graph with a logarithmic scale and... <ul style="list-style-type: none"> ○ 3 to 5 mistakes in the table ○ The shape of the graph is approximately correct • Exponential Graph or pH graph ($pH = -\log[H^+]$) <ul style="list-style-type: none"> ○ 1 to 3 mistakes in the table ○ Graph is approximately correct
3	Developed <ul style="list-style-type: none"> • A logarithmic graph or linear graph with a logarithmic scale and... <ul style="list-style-type: none"> ○ Table with 1 or 2 mistakes ○ All coordinates graphed are correct ○ Scale is assumed • An exponential graph or pH graph ($pH = -\log[H^+]$) <ul style="list-style-type: none"> ○ Correct table ○ Coordinates graphed correctly ○ Scale provided if not scaling by ones
4	Advanced <ul style="list-style-type: none"> • A logarithmic graph or linear graph with a logarithmic scale and... <ul style="list-style-type: none"> ○ Correct table ○ Coordinates graphed correctly ○ Scale provided if not scaling by ones

Note: Logarithmic scale means that each tic mark on the axis is a power of 10.

MG3:

What are the domain, asymptote, x-intercept, and y-intercept of the following function?

$$f(x) = \log_4(x - 1)$$

Domain: $(1, \infty)$ Asymptote: $x = 1$

x-intercept: 2

y-intercept: N/A

Score	Description
0	<ul style="list-style-type: none">• No response• Incorrect response
1	Undeveloped <ul style="list-style-type: none">• One characteristic correct
2	Applied <ul style="list-style-type: none">• Two characteristics correct
3	Developed <ul style="list-style-type: none">• Three characteristics correct
4	Advanced <ul style="list-style-type: none">• Completely correct

SW1

Define or explain the pH value.

Level 4: Park and Choi (2012) states that a student with an advanced level of understanding of pH will define it as a measure of the hydrogen ion concentration.

Level 3: Park and Choi state that a student with a type 3 operation level of understanding of pH could define pH as the $-\log [H^+]$. Though the formula is the mathematical definition of pH, the researchers state simply writing the formula is not an indication of an advanced level of understanding. In addition, a student that refers to pH as a measure of hydrogen and not hydrogen ion concentration indicates a functional relationship for pH, but it is unclear if they understand that pH is a measure of hydrogen ion concentration

Level 2: Park and Choi state that a student at a type 2 operational level can calculate values of pH given a hydrogen ion concentration. However, the student calculates the value from memorization and not logarithmic understanding. Students at this level of understanding could provide an example of the pH of a given concentration

Level 1: Park and Choi determined that students at the lowest level of understanding will define pH as a measure of acidity or as a way to classify a substance as an acid, base, or neutral. At this level, students treat pH as a characteristic of the substance and not as a function of hydrogen ion concentration.

SW1

Score	Description
0	No response, unrelated definition
1	Undeveloped: A student understands pH as a measure of acidity, and the pH value is treated as a characteristic of the substance and not a variable. <ul style="list-style-type: none"> • A student refers to pH as a measure of acidity. • A student provides a range for pH (0 to 6 acid, 7 neutral, 8-14 base)
2	Applied: A student understands that pH is a measure of acidity and can use basic mathematical operations (add, subtract, multiply, and divide) or uses memorized facts to evaluate pH. <ul style="list-style-type: none"> • A student provides an example of pH with a corresponding concentration • A student refers to pH as the exponent • A student refers to calculating pH using a double number line
3	Developed: A student understands pH as a measure of acidity and can use logarithms to describe pH OR a student understands pH as a measure of the concentration of Hydrogen ions, but uses basic mathematical operations to describe pH. <ul style="list-style-type: none"> • A student writes the formula for pH • A student states that pH is a measure of hydrogen or hydrogen ions, not concentration of hydrogen ions

	<ul style="list-style-type: none"> A student provides a mathematical definition such as pH is the adding world or pH is the corresponding exponent for a given argument (not in the context of chemistry, but correct)
4	<p>Advanced: A student understands pH as a measure of the hydrogen ion concentration and can use logarithms to describe pH</p> <ul style="list-style-type: none"> A student refers to pH as the “measure of the concentration of hydrogen ions”

SW2

Explain the meaning in context of pH for the following using complete sentences.

$$-\log(1 \times 10^{-7})$$

Level 4: Park and Choi (2012) state that a student at the type 5 function level will calculate the pH value as seven and state the meaning of 1×10^{-7} as the hydrogen ion concentration. If a student classifies the pH value as a neutral solution without stating that the pH equals seven, then an understanding of the pH value of seven can be assumed.

Level 3: A student at the Type 3 operation level will be able to calculate the pH value of seven, and a student at the type 4 function level will understand that the expression represents the relationship between a specific hydrogen ion concentration and a pH value but he or she will not be able to calculate that value (Park & Choi, 2012). A student that shows a developed level of understanding of this problem will be able to calculate the logarithm but provide no further explanation, or they will understand that the expression is the relationship between a specific hydrogen ion concentration and pH, but not be able to evaluate the expression.

Level 2: A student at an applied level of understanding would be expected to state only that the mathematical expression is pH or only that 1×10^{-7} is a hydrogen ion concentration, but not be able to perform the operation.

Level 1: A student at an undeveloped level of understanding could simply state that it will only be the pH of a solution or that it will measure whether the solution is an acid or a base.

SW2

Score	Description
0	No response, unrelated explanation
1	<p>Undeveloped: A student understands pH as a measure of acidity, and the pH value is treated as a characteristic of the substance and not a variable.</p> <ul style="list-style-type: none"> • A student restates the problem correctly • A student makes a general statement that could be considered correct such as it is an acid, base, or neutral solution. • A student confuses 1×10^{-7} for pH
2	<p>Applied: A student understands that pH is a measure of acidity and can use basic mathematical operations (add, subtract, multiply, and divide) to describe pH or uses memorized facts to evaluate pH.</p> <ul style="list-style-type: none"> • A student only states that it will tell whether it will be an acid or a base or that it will be the pH value of the solution (Student does not calculate the value) • A student only indicates that 1×10^{-7} is the $[H^+]$ • A student provides only an estimated value for pH • A student refers to it as an exponent • A student only provides 7 with no explanation or that the exponent is -7 • A student states that the answer will be positive
3	<p>Developed: A student understands pH as the measure of acidity and can use logarithms to describe pH OR a student understands pH as a measure of the concentration of Hydrogen ions but uses basic mathematical operations to describe pH.</p> <ul style="list-style-type: none"> • A student only states that the substance has a pH of 7 • A student indicates that 1×10^{-7} is the hydrogen ion concentration of the solution and indicates that the resulting value is pH, but the value is incorrect, estimated, or not provided • A student explains how to evaluate the problem mathematically using a double number line or properties of logarithms without providing the pH value of 7 • A student explains that it will be the exponent that you raise 10 to get 1×10^{-7}
4	<p>Advanced: A student understands pH as a measure of the hydrogen ion concentration and can use logarithms to describe pH</p> <ul style="list-style-type: none"> • A student states that the pH of the solution with a hydrogen ion concentration of 1×10^{-7} is equal to 7 or the solution is neutral.

SW3

Is it possible to have hydrogen ion concentration $[H^+]$ of 0? Why or Why not?

Level 4: A student at an advanced level of understanding should state that the hydrogen ion concentration cannot be zero and provides a mathematical or scientific reason. For example, a student could state that the hydrogen ion concentration is a power of 10 and will never reach zero. In addition, a student could state that ionization constant of water is a non-zero number and the product of the hydrogen ion and hydroxide ion concentrations.

Level 3: A student at a developed level shows a functional level of understanding by correctly linking the hydrogen ion concentration to pH (Park & Choi, 2012). However, the student indicates that their knowledge of pH is limited to the typical range zero to fourteen.

Level 2: A student at the applied level of understanding could make a statement that indicates a memorized response or treats the relationship between pH and hydrogen ion concentration as a linear difference (Park & Choi, 2012). Therefore, a student could state that the hydrogen ion concentration cannot be zero. Even though the student is restating the problem, the response indicates a memorized fact. In addition, the student can link the hydrogen ion concentration to pH but provide an incorrect answer or explanation.

Level 1: A student at an undeveloped level of understanding will answer this question with an answer that is related but incorrect. Park and Choi (2012) state that a student at the object level of understanding treats pH as a characteristic of a substance or a value that has no meaning beyond classifying solutions as acids or bases. Therefore, students at the object level may also treat the hydrogen ion concentration in the same way. These students may respond with a reason such as hydrogen is in the chemical formula of water.

SW3

Score	Description
0	<p>No response, unrelated explanation, or no explanation</p> <ul style="list-style-type: none"> • Yes or no without an explanation • A student states it would be negative • A student states that there is no answer that is zero
1	<p>Undeveloped: A student understands pH as a measure of acidity, and the pH value is treated as a characteristic of the substance and not a variable.</p> <ul style="list-style-type: none"> • Yes or No with a related but incorrect explanation. <ul style="list-style-type: none"> ○ Hydrogen is in water ○ Zero means you have nothing
2	<p>Applied: A student understands that pH is a measure of acidity and can use basic mathematical operations (add, subtract, multiply, and divide) to describe pH or uses memorized facts to evaluate pH.</p> <ul style="list-style-type: none"> • No, pH cannot go that high • No, because hydrogen ion concentration or it cannot be zero • No, and a student make an incorrect link to pH such as $\text{pH} + \text{pOH} = 14$ therefore it can be zero if the pOH is 14 • Yes, and the student confuses pH and $[\text{H}^+]$ and calculates the corresponding $[\text{H}^+]$ for a pH of 0. • Yes, and the student states that the substance needs to be completely basic
3	<p>Developed: A student understands pH as a measure of acidity and can use logarithms to describe pH OR a student understands pH as a measure of the concentration of Hydrogen ions but uses basic mathematical operations to describe pH.</p> <ul style="list-style-type: none"> • No, and a student correctly links it to pH or measuring acids and bases, but their knowledge is limited to the typical pH scale.
4	<p>Advanced: A student understands pH as a measure of the Hydrogen ion concentration and can use logarithms to describe pH</p> <ul style="list-style-type: none"> • No, and the student links the hydrogen ion concentration to a base 10 system (or multiplying world), to the ionization constant of water, or to the disassociation of water molecules.

SW4

Explain the range of pH values? (You can explain by giving specific examples or descriptions)

Level 4: A student at an advanced level of understanding will understand the logarithmic relationship between the hydrogen ion concentration and pH values (Park & Choi, 2012). Therefore, the students will understand that pH can extend beyond the typical range of 0 to 14.

Level 3: A student at a developed level of understanding will know that there is a relationship between hydrogen ion concentration and pH, but he or she will lack the knowledge to extend the pH values beyond the typical 0 to 14 range. In addition, these students will indicate a generalization level of understanding, because they will indicate that pH value can be non-integer numbers (Weber, 2002).

Level 2: A student at the applied level of understanding will know the typical range for pH, but will not understand that pH can be non-integer values

Level 1: A student at the lowest level of understanding of pH will only be able to state that pH is a measure of acidity or provide specific examples of solutions and their corresponding pH values (Park & Choi, 2012). Students will treat pH as a characteristic and not a logarithmic relationship.

SW4

Score	Description
0	<p>No response, unrelated explanation, or no explanation</p> <ul style="list-style-type: none"> A student only gives whole number values (no interval provided) or examples
1	<p>Undeveloped: A student understands pH as a measure of acidity, and the pH value is treated as a characteristic of the substance and not a variable.</p> <ul style="list-style-type: none"> A student understands that the pH can be 0 to 14 or 1 to 14 A student states that the range of the hydrogen ion concentration is (0 to infinity) and provides no justification. A student states negative infinity to infinity with no explanation
2	<p>Applied: A student understands that pH is a measure of acidity and can use basic mathematical operations (add, subtract, multiply, and divide) to describe pH.</p> <ul style="list-style-type: none"> A student states that the typical range is 0 to 14 or 1 to 14, and acids have a pH from 0 or 1 to 6 and bases have a pH of 8 to 14 A student provides the typical range and states that the lower the pH the more acidic and the higher the pH the more basic. A student states that the typical range is 0 to 14 or 1 to 14, and 7 is neutral. No other information is given.
3	<p>Developed: A student understands pH as a measure of acidity and can use logarithms to describe pH OR a student understands pH as a measure of the concentration of hydrogen ions but use basic mathematical operations to describe pH.</p> <ul style="list-style-type: none"> A student states the range is from 0 to 14, pH values less than 7 are acidic, pH values greater than 7 are basic, and a pH value of 7 is neutral. A student states 0 to 14 or 1 to 14 and indicates that it is a measure of hydrogen ion concentration. A student state that according to the mathematical formula the range is negative infinity to infinity.
4	<p>Advanced: A student understands pH as a measure of Hydrogen ion concentration and can use logarithms to describe pH</p> <ul style="list-style-type: none"> A student states that the typical pH range is from 0 to 14 but indicates that it could be higher than 14 and lower than 0.

SG2

HCl solutions in seven different molarities were obtained from a chemistry laboratory.

HCl (hydrochloric acid) aqueous solution: 0.1, 0.02, 0.001, 0.0002, 0.00002, 0.000001, 0.0000002 M.

Display the relationship between molarities of HCl and pH values in the Cartesian coordinates below.



Level 4: For students to show that they have an advance knowledge of the relationship between pH and hydrogen ion concentrations, the students would have to show a “Generalization” level of understanding of logarithms (Weber, 2002) and a type 5 functional level of pH (Park & Choi, 2012) for this problem. Therefore, a student that can estimate the corresponding pH values for some of the given concentrations and graph an exponentially decaying function show the highest level of understanding required for this problem.

Level 3: For a student to show a level 3 understanding, he or she would have to demonstrate a “Generalization” level of understanding of logarithms and a type 4 functional level of understanding of pH. This student would be able to estimate the corresponding pH value for some of the given concentrations. However, the student would graph a linear decreasing function, because he or she does not have an understanding of the relative differences among the hydrogen ion concentrations.

Level 2: For a student to show a level 2 understanding, he or she would have to demonstrate a “Generalization” level of understanding of logarithms or an “operational” level of understanding of pH. This student would be able to estimate corresponding pH value but graph the function as increasing or graph a decreasing exponential function, but not be able to calculate corresponding pH values.

Level 1: A student that demonstrates an “Exponentiation as a process” level (Weber, 2002) shows the lowest level of understanding that can be indicated by this problem. This student would understand that as pH increases the hydrogen ion concentration decreases. The students could indicate this understanding by drawing a linear decreasing function.

Score	Description
0	<ul style="list-style-type: none"> No response or unrelated response An increasing linear function with no axes labeled or labeled incorrectly An increasing logarithmic function with no axes labeled or labeled incorrectly
1	<p>Undeveloped: A student understands pH as a measure of acidity, and the pH value is treated as a characteristic of the substance and not a variable.</p> <ul style="list-style-type: none"> A linear decreasing function and no corresponding pH values are calculated for the given concentrations.
2	<p>Applied: A student understands that pH is a measure of acidity and can use basic mathematical operations (add, subtract, multiply, and divide) to describe pH.</p> <ul style="list-style-type: none"> Only an exponential decay or growth graph is shown <ul style="list-style-type: none"> No scale on axes or scale and graph do not match No calculations of pH value of the various concentrations of HCl. 1 to more concentrations and the corresponding pH values are shown, but the relative differences between values are not scaled properly. (The graph is a linear increasing function)
3	<p>Developed: A student understands pH as a measure of acidity and can use logarithms to describe pH OR a student understands pH as a measure of the concentration of hydrogen ions but uses basic mathematical operations to describe pH.</p> <ul style="list-style-type: none"> 1 to more concentrations and the corresponding pH values are shown, but the relative differences between values are not scaled properly. (The graph is a linear decreasing function) 1 to 3 concentrations and the corresponding pH values are shown, and the graph is an exponential decay and no scale is provided on the graph. All concentrations and the corresponding pH values are shown, and the graph is not shown or not scaled
4	<p>Advanced: A student understands pH as a measure of Hydrogen ion concentration and can use logarithms to describe pH</p> <ul style="list-style-type: none"> Some concentrations and the corresponding pH values are shown, and the relative differences between the values are scaled and labeled properly (The graph is an exponential decay).

SG3

Provide an interval (lowest possible to highest possible) for hydrogen ion concentration and pH value.

Hydrogen ion:

pH value:

Score	Description
0	<ul style="list-style-type: none"> No response or unrelated response
1	<p>Undeveloped: A student understands pH as a measure of acidity, and the pH value is treated as a characteristic of the substance and not a variable.</p> <ul style="list-style-type: none"> A student provides whole number values within the typical range, but does not provide an interval A student provides an example such as water has a pH of 7
2	<p>Applied: A student understands that pH is a measure of acidity and can use basic mathematical operations (add, subtract, multiply, and divide) to describe pH.</p> <ul style="list-style-type: none"> Typical range for pH (0 to 14 or 1 to 14) and range for concentration is not listed or not correct Typical range for concentrations (10^0 to 10^{-14} or 10^{-1} to 10^{-14}) and pH range is not listed or not correct The mathematical range for pH or concentration is given and the other is not listed or incorrect An interval within the typical range such as 1 to 7 and the corresponding concentration is provided.
3	<p>Developed: A student understands pH as a measure of acidity and can use logarithms to describe pH OR a student understands pH as a measure of the concentration of hydrogen ions but uses basic mathematical operations to describe pH.</p> <ul style="list-style-type: none"> The typical range for pH and concentration are given and no indication that the pH value can be negative or more than 14 and the interval for hydrogen ion concentration corresponds to the pH range. The mathematical range is given (pH: -infinity to infinity and $[H^+]$: 0 to infinity) and no further explanation indicating that pH and $[H^+]$ have a limit.
4	<p>Advanced: A student understands pH as a measure of the hydrogen ion concentration and can use logarithms to describe pH</p> <ul style="list-style-type: none"> A student indicates that the pH and $[H^+]$ can within reason be outside of the typical range. A student could state that pH can range from a little less than 0 to a little more than 14 and the $[H^+]$ can be a little more than 1×10^0 and a little less than 1×10^{-14}

Appendix L

Quantitative Analysis on the LPA

The Logarithms and pH assessment instrument (LPA) was used to compare the control ($n = 17$) and treatment groups ($n = 23$) on their conceptual understanding of logarithms in both a mathematics context and science context. The differences in the control and treatment groups scores on their conceptual understanding of logarithms in a mathematics context was measured by comparing the scores of the CU factor and the CW and CN sub factors from the mathematics portion of the assessment instrument. In addition, the differences in the control and treatment groups scores on their conceptual understanding in a science context was measured by comparing the SN and SW components as well as the overall science score (SO). To compare the scores, the data was first checked against several assumptions before the type of analysis was chosen. Once the assumptions were checked, a statistical procedure was chosen to provide an inference about the null hypothesis (H_0) that there is no statistically significant difference in the scores on the LPA factors between groups and alternative hypothesis (H_a) that there is a statistically significant difference ($p < 0.05$) in the scores. After the analysis, the findings were interpreted in the context of this study.

The study required that the control and treatment groups be compared to determine if there exist a statistically significant difference between the groups on factors from the LPA. To determine the appropriate statistical analysis, the researcher checked several assumptions. First, the students' scores can be considered independent because the control group and treatment group were in separate algebra II and chemistry courses and took the test individually and received no help from the instructors in the room. Therefore, an independent sample t-test or a

Mann-Whitney U test were the most appropriate statistical test. Second, the researcher collected the data from the LPA and analyzed it for normality using the Shapiro-Wilk test of Normality (Table 14) to determine if the t-test or Mann-Whitney U test should be used. Because most of the data were considered non-parametric, the researcher used the Mann-Whitney U test in the analysis of the LPA. Third, the researcher compared the data to determine if the data distribution of both groups had similar shapes using the Levene's test of homogeneity of variances (Table 15). The data proved to be mostly non-parametric and have similar distribution. Therefore, a Mann-Whitney U test was conducted on each factor and sub-factor to determine if there exist a statistically significant difference between the mean ranks of the control and treatment groups.

Table 14

Test of Normality on LPA Factors

		Shapiro-Wilk		
	Group	Statistic	df	Sig.
CW	Treatment	.879	23	.010
	Control	.978	17	.932
CN	Treatment	.861	23	.004
	Control	.809	17	.003
CU	Treatment	.929	23	.104
	Control	.969	17	.805
SW	Treatment	.962	23	.506
	Control	.921	17	.155
SN	Treatment	.866	23	.005
	Control	.787	17	.001
SO	Treatment	.971	23	.709
	Control	.912	17	.109

Table 15

Test of Homogeneity of Variances on LPA Factors

		Levene Statistic	df1	df2	Sig.
CW	Based on Mean	.808	1	38	.374
	Based on Median	1.332	1	38	.256
	Based on Median and with adjusted df	1.332	1	37.908	.256
	Based on trimmed mean	.946	1	38	.337
CN	Based on Mean	1.182	1	38	.284
	Based on Median	.924	1	38	.342
	Based on Median and with adjusted df	.924	1	37.756	.342
	Based on trimmed mean	1.056	1	38	.311
CU	Based on Mean	.193	1	38	.663
	Based on Median	.102	1	38	.751
	Based on Median and with adjusted df	.102	1	37.503	.751
	Based on trimmed mean	.182	1	38	.672
SW	Based on Mean	4.022	1	38	.052
	Based on Median	4.196	1	38	.047
	Based on Median and with adjusted df	4.196	1	35.041	.048
	Based on trimmed mean	4.113	1	38	.050
SN	Based on Mean	2.297	1	38	.138
	Based on Median	.314	1	38	.579
	Based on Median and with adjusted df	.314	1	31.595	.579
	Based on trimmed mean	1.780	1	38	.190
SO	Based on Mean	.065	1	38	.800
	Based on Median	.000	1	38	.984
	Based on Median and with adjusted df	.000	1	37.701	.984
	Based on trimmed mean	.046	1	38	.831

The factors CW, CN, and CU were compared using a Mann-Whitney U Test. From the test, there existed a statistically significant difference on the CU factor. The test indicated that CU was greater for the control (Mdn = 4.5) than for the treatment (Mdn = 2.5), $U = 122$, $p = 0.045$. In addition, the Mann-Whitney U test on the sub-factor CW indicated that CW was greater for the control (Mdn = 5) than for the treatment (Mdn = 3), $U = 122$, $p = .044$. Therefore, a statistically significant difference between the control and treatment group exist on the CU and CW factors. This result indicated that the integrated STEM unit on logarithms and pH did not develop a student's level of conceptual understanding of logarithms in a mathematics context as well as the traditional unit as measured by the LPA, which defines conceptual understanding as flexible knowledge that can be applied to multiple types of problems within a context (Panasuk, 2010; Rittle-Johnson et al., 2001). However, the Mann-Whitney U test on the sub-factor CN indicated that the difference between the control (Mdn = 2) and the treatment (Mdn = 1) was not statistically significant, $U = 129.5$, $p = 0.054$. This result indicated that the integrated STEM approach equally develops the conceptual understanding of the numerical representation of logarithms in a mathematics context.

The factors SW, SN, and SO were compared using a Mann-Whitney U Test. The test on the factor SW indicated that the difference between the control (Mdn = 7) and treatment (Mdn = 7) was not statistically significant, $U = 184$, $p = .752$. However, the Mann-Whitney U test on the factor SN indicated that a statistically significant difference exists between the groups. The control group (Mdn = 3) had scores statistically greater than the treatment (Mdn = 2), $U = 109.5$, $p = 0.017$. The test on SO indicated that there was not a statistically significant difference between the control (Mdn = 4.75) and the treatment (3.5), $U = 141$, $p = 0.141$. The results indicated that students in the traditional classroom setting perform better on numerical

calculations of pH and hydrogen ion concentrations, and that students in either setting will be able to communicate their understanding of pH through writing. However, the overall conceptual understanding of logarithms in a science context was not significantly different.

The effect sizes of the statistically significant results on the CW, CU, and SN factors are considered small ($\eta^2 < 0.20$) (Lenhard & Lenhard, 2016). The effect size on the CW factor indicated that 11% of the variability in the ranks is accounted for by the group that a student participated. The effect size on the CU factor indicated that approximately 10% of the variability in the ranks is accounted for by the group that a student participated. In addition, the effect size on the SN factor indicated that approximately 15% of the variability in the ranks is accounted for by the group that a student participated.